STACKS

SOCIAL SCIENCES AND EDUCATION

NAVAL RESEARCH LOGISTICS QUARTERLY

OFFICE OF NAVAL RESEARCH

CONTENTS

ARTICLES	Page
The Interdependency of Logistic and Strategic Planning *Captain S. E. Ruehlow*	237
Mobile Logistics Support in the "Passage to Freedom" Operation *Commander Randolph Meade, Jr., and Lt. Commander Carl A. Fischer*	258
Consistency Problems in the Military Supply System *Oskar Morgenstern*	265
The Cargo-Handling System *R. R. O'Neill and J. K. Weinstock*	282
A Scientific Method for Establishing Reorder Points *Morris J. Solomon*	289
The Theory of Inventory Management—A Review *Thomas V. V. Atwater, Jr.*	295
Minimization of Non-Linear Separable Convex Functionals *A. Charnes and C. E. Lemke*	301
Fictitious Play for Continuous Games *J. M. Danskin*	313
An Elementary Treatment of an Input-Output System *Y. K. Wong*	321
Decision Making in the Face of Uncertainty - II *Richard Bellman*	327
NEWS AND MEMORANDA	333
RECENT PUBLICATIONS	336

Los Angeles Public Library
AUG 5 1955
DEPOSITORY

VOL. 1, NO. 4 — DECEMBER 1954

NAVEXOS P-1278

NAVAL RESEARCH LOGISTICS QUARTERLY

EDITORS

Rear Admiral H. E. Eccles, USN (Retired)
The George Washington University
Washington 7, D.C.

O. Morgenstern
Princeton University
Princeton, N. J.

Commander G. S. Essenwine, USN
Office of Naval Research
Washington 25, D. C.

F. D. Rigby
Office of Naval Research
Washington 25, D. C.

J. Laderman, *Managing Editor*
Office of Naval Research
Washington 25, D. C.

ASSOCIATE EDITORS

D. Blackwell, *Howard University*
R. M. Bowstrom, *Captain, SC, USN*
E. W. Cannon, *National Bureau of Standards*
G. B. Dantzig, *The Rand Corporation*
M. M. Flood, *Columbia University*
J. D. Hayes, *Rear Admiral, USN (Retired)*
E. B. Henry, Jr., *Commander, USN*
A. T. Magnell, *Captain, SC, USN*
J. Marschak, *University of Chicago*
T. C. Roberts, *Commander, SC, USN*
E. K. Scofield, *Commander, SC, USN*
E. D. Stanley, *Captain, SC, USN*
H. Strock, *Commander, SC, USN*
R. M. Thrall, *University of Michigan*
C. B. Tompkins, *University of California*
A. W. Tucker, *Princeton University*
R. E. Williams, *Commander, SC, USN*
M. A. Woodbury, *The George Washington University*

The Naval Research Logistics Quarterly is devoted to the dissemination of scientific information in logistics and will publish research and expository papers, including those in certain areas of mathematics, statistics, and economics, relevant to the over-all effort to improve the efficiency and effectiveness of logistics operations.

Manuscripts submitted for publication should be typewritten, double-spaced, and the author should retain a copy. Manuscripts and other items for publication should be sent to the Managing Editor at the above address. Authors will receive 50 free reprints of their published papers.

The Naval Research Logistics Quarterly is published by the Office of Naval Research in the months of March, June, September, and December and can be purchased from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Subscription Price: $1.50 a year in the U. S. and Canada, $2.00 elsewhere; 50¢ for a single copy. Reprints of an individual article can be purchased in quantities of 100 or more. Requests for the purchase price of reprints of a particular article should be sent to the Superintendent of Documents.

The views and opinions expressed in the articles in this Quarterly are those of the authors and not necessarily those of the Office of Naval Research.

The printing of this publication was approved by the Director of the Bureau of the Budget, June 30, 1954.

CONTENTS OF VOLUME 1

ARTICLES	Pages
ANDERSON, Rear Admiral Bern (Retired). A New Field for Logistics Research	79-81
ATWATER, Thomas V. V., Jr. The Theory of Inventory Management – A Review	295-300
BELLMAN, Richard. On Some Applications of the Theory of Dynamic Programming to Logistics	141-153
BELLMAN, Richard. Decision Making in the Face of Uncertainty – I	230-232
BELLMAN, Richard. Decision Making in the Face of Uncertainty – II	327-332
BLACKETT, D. W. Some Blotto Games	55-60
BLACKWELL, David. On Multi-Component Attrition Games	210-216
CHARNES, A., and C. E. Lemke. Minimization of Non-Linear Separable Convex Functionals	301-312
DANSKIN, J. M. Fictitious Play for Continuous Games	313-320
DANTZIG, G. B., and D. R. Fulkerson. Minimizing the Number of Tankers to Meet a Fixed Schedule	217-222
ECCLES, Rear Admiral Henry E., (Retired). Logistics—What Is It?	5-15
ECCLES, Rear Admiral Henry E., (Retired). The Rommel Papers—A Commentary	103-108
ECCLES, Rear Admiral Henry E., (Retired). The Quartermaster Corps: Organization, Supply and Services, Volume I—A Book Review	207-209
ENGLISH, Captain James A., and E. A. Jerome. Statistical Methods for Determining Requirements of Dental Materials	191-199
FISCHER, Lt. Commander Carl A., and Commander Randolph Meade, Jr. Mobile Logistics Support in the "Passage to Freedom" Operation	258-264
FULKERSON, D. R., and G. B. Dantzig. Minimizing the Number of Tankers to Meet a Fixed Schedule	217-222
GADDUM, J. W., A. J. Hoffman, and D. Sokolowsky. On the Solution of the Caterer Problem	223-229
GAINEN, Leon, Commander E. D. Stanley, and Lieutenant D. P. Honig. Linear Programming in Bid Evaluation	48-54
HAYES, Rear Admiral John D., (Retired). Logistics—The Word	200-202
HETTER, Rear Admiral F. L. Navy Stratification and Fractionation for Improvement of Inventory Management	75-78
HOFFMAN, A. J., J. W. Gaddum, and D. Sokolowsky. On the Solution of the Caterer Problem	223-229
HOLLINGSHEAD, E. F. The Application of Statistical Techniques to Management of Overseas Supply Operations	82-89
HONIG, Lieutenant D. P., Commander E. D. Stanley, and Leon Gainen. Linear Programming in Bid Evaluation	48-54

ARTICLES

	Pages
HORNE, R. C. Developing an Engineered Productivity Standard	203-206
JACOBS, Walter. The Caterer Problem	154-165
JEROME, E. A., and Captain James A. English. Statistical Methods for Determining Requirements of Dental Materials	191-199
JOHNSON, S. M. Optimal Two- and Three-Stage Production Schedules with Setup Times Included	61-68
LEMKE, C. E. The Dual Method of Solving the Linear Programming Problem	36-47
LEMKE, C. E., and A. Charnes. Minimization of Non-Linear Separable Convex Functionals	301-312
LYNCH, Captain Charles F. Notes on Applied Analytical Logistics in the Navy	90-102
MARSCHAK, J., and M. R. Mickey. Optimal Weapon Systems	116-140
MEADE, Commander Randolph, Jr., and Lt. Commander Carl A. Fischer. Mobile Logistics Support in the "Passage to Freedom" Operation	258-264
MICKEY, M. R., and J. Marschak. Optimal Weapon Systems	116-140
MORGENSTERN, Oskar. Consistency Problems in the Military Supply System	265-281
O'NEILL, R. R., and J. K. Weinstock. The Cargo-Handling System	282-288
RUEHLOW, Captain S. E. The Interdependency of Logistic and Strategic Planning	237-257
ST. JOHN, Lieutenant Colonel Lawrence R. Trends in Logistics	182-190
SOKOLOWSKY, D., A. J. Hoffman, and J. W. Gaddum. On the Solution of the Caterer Problem	223-229
SOLOMON, Morris J. A Scientific Method for Establishing Reorder Points	289-294
STANLEY, Commander E. D., Lieutenant D. P. Honig, and Leon Gainen. Linear Programming in Bid Evaluation	48-54
von NEUMANN, John. A Numerical Method to Determine Optimum Strategy	109-115
WEINSTOCK, J. K., and R. R. O'Neill. The Cargo-Handling System	282-288
WHITIN, T. M. On the Span of Central Direction	25-35
WILLIAMS, Commander Ralph E., Jr. Some Thoughts on Logistic Planning Factors	173-181
WONG, Y. K. An Elementary Treatment of an Input-Output System	321-326
YOUNG, William M. Priorities in the Naval Supply System	16-24

MISCELLANEOUS

A Message from Rear Admiral F. R. Furth	1
Preface	3-4
News and Memoranda	69-71, 166-168, 233-234, 333-335
Recent Publications	72-74, 169-171, 235-236, 336-338

THE INTERDEPENDENCY OF LOGISTIC AND STRATEGIC PLANNING[1]

Captain S. E. Ruehlow
United States Navy

This paper contains the following sections:

INTRODUCTION

I. THE INFLUENCE OF LOGISTICS ON STRATEGIC PLANNING IN WORLD WAR II.

A. The Status of Logistic Planning at the Beginning of World War II.
B. Concurrent Planning.
C. The International Conference at Cairo.

II. THE PLANNING RESPONSIBILITIES OF THE JOINT CHIEFS OF STAFF.

III. INTEGRATION OF CURRENT STRATEGIC AND LOGISTIC PLANNING.

A. Types of Joint Plans.
B. Joint Chiefs of Staff Program for Planning.

IV. THE DEVELOPMENT OF REQUIREMENTS.

A. Purpose.
B. Methods.
C. Evaluation of Methods.

SUMMARY
CONCLUSIONS
APPENDIXES
BIBLIOGRAPHY

INTRODUCTION

1. A major war of the future will be, in all probability, a total war involving entire civilian populations and will require the fullest utilization of the resources and the productive capacity of the nations involved. Wars become progressively more expensive in terms of expenditure of resources and the destruction of capital assets. Another war will be so expensive as to be a threat to the national economy. Resources are not unlimited or inexhaustible and a victory would prove to be an empty triumph if the nation had expended its resources and economic assets in achieving it.

2. From World War II, we learned, among other things, that there are certain deficiencies in resources to fight a global war. Manpower and productive capacity proved to be limited for waging what might be called a conventional type war and serious problems in production and distribution faced the civilian and military planner. Although the same problems faced the nation in wars of the past, in varying degrees, previously the time available for mobilization has not been critical. Time will be a critical factor in a future war and the problems of mobilization in this age of advanced technology will be infinitely more complex. The effectiveness of current mobilization and logistic planning for a possible war may be a determining factor in the outcome of such a war.

[1]This article was prepared as a term paper at the Industrial College of the Armed Forces in May 1954.

The recognition of this factor has lead to a new awareness of the vital need for realistic mobilization planning and concurrent logistic and strategic planning.

3. The purpose of this report is to discuss the interrelationship of logistic and strategic planning and the progress that has been achieved in integrating logistic and strategic planning since World War II. The development of logistic planning will be discussed, drawing on World War II experience and post war developments. The report includes a discussion of the Joint Chiefs of Staff and Office of Defense Mobilization programs for mobilization planning, the determination of military and civilian requirements, the testing of strategic war plans for feasibility and the resultant courses of action which may be indicated. It is felt that only through a thorough understanding of the purpose of these phases of planning and the need therefor can a clear understanding of the interdependency of logistic and strategic planning be realized.

I. THE INFLUENCE OF LOGISTICS ON STRATEGIC PLANNING IN WORLD WAR II.

A. The Status of Logistic Planning at the Beginning of World War II.

1. Prior to World War II, the military, as well as non-military, were apt to look back on World War I as a big war and one that was about as extended as war could become. Looking back now on World War I, one is likely to be somewhat surprised to be reminded that it was, for all practical purposes, a one theater war with one major pipeline of supply and with one major command in the field to which all other efforts were subordinated. Logistic planning was relatively simple since planning for production, supply and distribution could be directed at a single objective and had to take into account the minimum of alternate strategies. Consequently, the importance of logistic planning and logistic planners was not felt and furthermore, the need for sound and realistic logistic plans to support strategy had not been recognized, as yet, at the beginning of World War II. Up to this time, the imagination and the ideas of the operational and strategic planners were considered paramount and all that was necessary. Little if any attention or thought was given to the logistic planner whose job it was to carry out the support of these strategic plans.[2]

2. When World War II was forced upon us, we found that instead of one major theater of operations, there were eleven major theaters spread completely around the globe, none of which could be safely neglected or left inactive. The United States was not prepared to fight this type of a war, either from the standpoint of war plans or from the standpoint of availability of men, materials and equipment. It was found that the war plans which did exist were grossly infeasible because of the shortage of men, materials and shipping. The application of our limited stocks and limited war production to the demands of eleven theaters would have resulted in their ineffective dissipation. As a result, our efforts were directed towards defensive measures to protect our sea coasts and the Carribean and in other areas which were not combat areas, necessarily accepting the risk of possible further advance by the enemy in all theaters.

3. During the early days of World War II, the Joint Chiefs of Staff had only a staff of strategic planners to assist them. The officer in command of the Service Forces of the Army was the logistic representative and was largely responsible for logistic planning. His staff was neither adequate in size nor in training to cope with the task of the planning for and the integration of the requirements of the Army, Navy, Air Forces, shipping and all the other specialized

---

[2] Badger, Oscar C., VAdm, USN. The Influence of Logistics on Military Strategy. Newport R.I., Naval War College, 11 Dec. 1952. p. 2. (U 168 B2).

activities. For the first time, the logistic planner was faced with the problem of the timing of the execution of various operational plans so that such operations would not overlap and thus overload the logistics capacity. For instance, it became evident for the first time that the availability of shipping alone dictated the date of execution of operations. This required the closest possible coordination between strategic and logistic planners. As a matter of fact, it soon became clear that there had to be complete agreement between the two and the Joint Chiefs of Staff could not direct the execution of an operation until the logistic planners had determined that the operation was logistically feasible.

4. Until it was fully realized that the above principle was vital to the success of any war plan, mistakes were inevitably made which resulted in the ineffective application and loss of much of a very limited military force. The Joint Chiefs of Staff were quick to recognize the need for a logistic planning staff, however, and added to the Joint staff a group of Logistics Planners as a counterpart to the Strategic Planners. It was the responsibility of the Logistic Planners to test all operational plans for feasibility and to make recommendations to the Joint Chiefs of Staff as to whether such plans should be executed as scheduled or should be delayed. The Logistics Planners made many of the controlling recommendations governing the timing of operations in the various theaters and became a vital part of the Joint Chiefs of Staff organization.

B. Concurrent Planning.

1. Within the military, concurrent planning has come to mean the development of logistic plans concurrently with the development of strategic plans and close coordination of the two planning groups throughout the planning process. Prior to and during the early phases of World War II, there was a tendency for the strategic planners to hold their thoughts and information from those who had to work out the subordinate and logistic detailed plans. Concurrent planning is the only practical system for keeping strategic planning on the track and for expeditiously completing the system of subsidiary plans. It means that the logistic planners must be allowed to look over the shoulders of the strategic planners, to know their thoughts and to see the first drafts of tentative plans.[3]

2. By early 1942, the Combined Chiefs of Staff had developed a tentative grand global strategy and had agreed upon certain strategic concepts. However, no broad strategic plan had been developed upon which the logistical services of the Army and Air Force could base long range procurement requirements and supply plans. Although the Navy was somewhat better off with its unilateral plan for fleet expansion, guidance upon which civilian agencies such as the War Production Board, could base priority allocations and controls were lacking. In fact, without strategic plans, it was difficult to determine wherein industry should be expanded as no requirements, either for types or quantities of equipment, could be determined.

3. Time itself is of the essence to the logistic planner. The grand strategy of World War II was still under discussion as late as August 1942 when the invasion of North Africa was decided upon.[4] The logistic planners, knowing the length of time necessary to translate strategic plans into requirements and to produce the equipment, were in a very difficult position. The Army Service Force undertook the task of drawing up plans, themselves, to cover all possible

---

[3] Ibid. p. 9

[4] Lutes, LeRoy., LtGen, USA. Logistics in Grand Strategy. Wash., D.C., ICAF, 28 Feb. 1950. p. 12. (U 168 18).

operations in all possible theaters and to examine all the logistic factors involved. In effect, this logistical organization attempted to outguess the strategic planners in order to get production rolling.

4. In the early stages of broad planning, experienced and imaginative logistic planners can proceed with planning if they are informed of the areas in which operations will be conducted, the nature and magnitude of operations and alternate operations planned, and the approximate timing. Broad long-range logistic plans made on this basis must be refined as more detailed strategic and operational information is evolved. Such broad plans, while not providing for any specific operation, do provide for a reasonable procurement program from which early requirements may be determined and will point up prospective bottlenecks and extraordinary requirements, as well as provide a basis for determining priorities for mobilization and production in the early phases of a war.[5]

5. General Somervell, fortunately, had the courage to commence procurement and production on the basis of these plans. Although the reluctance of strategic planners to show their plans grew out of the fear that these plans would be considered firm approved plans, the logistic planners should never be placed in the position of having to develop strategic plans in order to initiate procurement. As stated previously, time itself is of the essence as strategic plans must be translated into production and procurement commitments and into plans to induct, equip and move large masses of troups and support them. The logistic planner, furthermore, needs more than just a strategic concept - he should have a phased operation plan.[6]

6. There are many examples of duplication of effort and reduction in effectiveness resulting from the lack of concurrent planning and the recognition on the part of strategic planners of the need for firm planning during the early phases of World War II. One striking example resulted from the indecision as to the plan of attack on North Africa. The logistic planners needed to know where shipping should be concentrated for combat loading. Although the target date was in October, and some units that were to be equipped were still being activated as late as September, the Army Service Force had not received a firm troop basis in August. As a result, the Army Service Force improvised a plan in order to prepare equipment for troops to be mounted from the United States. Furthermore, the thousands of tons of supplies and equipment which had been shipped to England in early 1942 for another operation were in a chaotic state and could not be identified and moved in time for equipping troops for the North African campaign. It was necessary to duplicate much of the equipment and ship it to England during October for loading from England for the assault convoy. Although these supplies and equipment were not lost, in the long run, the ineffective use of equipment "in being" detracted, time-wise, from the overall war effort. Much time as well as effectiveness would have been gained had the strategic and logistic planners worked side by side.[7]

C. The International Conference at Cairo.

1. The Joint and Combined Chiefs of Staff met in Cairo in late 1943 to lay plans for future operations in the eleven major theaters for the following year. The Theater Commanders of all eleven theaters had submitted their proposed courses of action for their own theaters in accordance with the established principles of decentralized planning. The strategic planners

---

[5]Esposito, V. J., Col., USA, Logistic Planning for War. Wash., D.C. ICAF, 4 Feb 1947, p. 6. (U 168 E8).

[6]Lutes, LeRoy, LtGen, USA, Logistics in Grand Strategy. Wash, D.C., ICAF, 28 Feb 1950. p. 3. (U 168 L8).

[7]Ibid. p. 7

had reviewed these plans and had evaluated them in the light of the military benefits to be derived and with respect to their contribution to global strategy. If it was determined that the execution of a plan would contribute to the early defeat of the enemy, the strategic planners recommended Joint Chiefs of Staff approval. Plans were laid at this conference for the invasion of Europe and the defeat of Germany and, in the Pacific, for all operations up to and including the capture of Okinawa.

2. Several theater operational plans were approved by the Joint Chiefs of Staff as being in consonance with and contributory to approved global strategy subject to the consideration of the logistic planners who would decide upon the feasibility and the timing of each plan.[8] These plans were:

a. The invasion of Normandy with 26 divisions on 4 May 1944.

b. The simultaneous invasion of southern France with 10 divisions.

c. Operations in Italy to continue at the then existing scale.

d. The invasion of Southern Europe through Greece and possibly through western Turkey with 14 divisions.

e. The logistic support of the Russians through the Persian Gulf.

f. The landing of 3 or 4 divisions at Moulmein between Siam and Malaysia to cut Japanese supply lines to Burma and Malaysia.

g. The invasion of the Philippines by several divisions under the command of General McArthur on 1 July 1944.

h. The capture of Guam and Saipan by forces under the command of Admiral Nimitz as bases for air operations in support of the invasion of the Philippines and the later capture of Okinawa.

3. The Joint Chiefs of Staff, having endorsed the above program, instructed the logistic planners to examine the plans and to make recommendations as to the feasibility of the overall program and as to best utilization of available manpower, industrial capacity and shipping. The logistic planning staff consisted of about 800 officers. The organization included personnel well versed in all of the many aspects of logistics including manpower, production capacity, shipping, petroleum and munitions. This staff was allowed 36 hours to study and evaluate the requirements of this extensive list of plans. The recommendations of the logistics planners were complete at the end of the 36 hours period. Several major changes to the general program were recommended as a result of this study. It was recommended that:

a. The invasion of Normandy be delayed until 4 June because the necessary men, equipment and shipping would not be available prior to that date. The major critical items of materials were steel plate, landing craft and landing craft engines.

b. The invasion of southern France be delayed 15 days until the dark of the moon because of insufficient air facilities available in the area and the resultant reduced air support for the operation.

c. The invasion of southern Europe through Greece to be cancelled based on a lack of sufficient landing craft and tankers, the neutrality of Turkey and the strength of German opposition in the area.

d. Although the voyage around South Africa meant a 135 day turn around time for each ship, which was about five times as great as the turn around time to Europe, the delivery of 5-1/2 million tons of supplies to the Russians through the Persian Gulf be approved. This

[8] Badger, op. cit., p. 4.

operation was approved on the basis that the supplies were well within our capacity and that the support of Russia on the Eastern Front would draw German divisions from the Western Front; thus improve the situation for the invasion of Normandy.

e. The invasion at Moulmein be cancelled because of a shortage of shipping for overall operations and the shortage of steel plate and landing craft. For political reasons, India refused to assist in furnishing steel, although she was capable of doing so. This action on the part of India influenced the final decision to abandon the operation.

f. The invasion of the Philippines by General MacArthur be delayed until after 10 October because of the lack of adequate shipping and production capacity to support both the landings in Normandy and the invasion of the Philippines, both of which were extensive operations and scheduled within a month of each other.

g. The advance of Admiral Nimitz's forces across the Pacific to be slowed somewhat because of the limited supply of aviation gasoline and tankers. This shortage resulted from the high requirements for the simultaneous operations to be conducted in western and southern France and the air operations from England.

4. The above historical summary of the planning effort for one year of actual combat operations is illustrative of the relationship between logistic planning and strategic planning in war and at a time when the nation was fully mobilized and producing at a near peak wartime rate. Because the logistic planners were thorougly familiar with the production potential and the availability of resources from constant association with the problem, they knew which items of materials and production facilities were in critical supply. When strategic plans were examined for feasibility, they were examined from the standpoint of the requirements for those resources which were known to be critical. This accounts for the ability of the logistic planners to test such extensive operations as were considered at Cairo in such a brief period of time. Only after the industrial and military potential of the nation has been fully mobilized and the potential is well known, is such a rapid feasibility test possible. At this particular time, there were about 10 known critical items including steel plate, shipping, landing craft, engines and aviation fuel. The extent of operations could be determined by estimating the availability of these few critical items for the period and the time span involved.

II. THE PLANNING RESPONSIBILITIES OF THE JOINT CHIEFS OF STAFF.

By the National Security Act of 1947, as amended in 1949, the Congress provided a comprehensive program for the security of the United States. Among other things, it provided for three military departments and a Joint Chiefs of Staff, a Chairman and a Joint Staff. The Congress also set forth the duties to be performed by the Joint Chiefs of Staff. These duties included but are not limited to those planning responsibilities indicated below.

1. The Joint Chiefs of Staff are responsible for the preparation of strategic plans and the strategic direction of the military forces. The Joint Strategic Plans Group of the Joint Staff is responsible for drawing-up such strategic plans as the Joint Chiefs of Staff may direct. When such plans have been reviewed by the services and by the Joint Strategic Plans Committee, they are sent to the Joint Chiefs of Staff for consideration and approval. When approved, the Joint Chiefs of Staff direct the issuance of the plans to the services and to other agencies in the Department of Defense in accordance with an approved distribution list. The plans then become the basis for service planning.

2. The Joint Chiefs of Staff are responsible for the preparation of joint logistic plans and the assignment to the military services of logistic responsibility in accordance with such

plans. Joint logistic plans are prepared by the Joint Logistics Plans Group of the Joint Staff and are reviewed by the services and by the Joint Logistics Plans Committee. Joint logistics plans are normally developed concurrently and in conjunction with the development of joint strategic plans. As in the case of the strategic plans, when approved by the Joint Chiefs of Staff, logistic plans are issued for planning guidance and direction.

3. It is the responsibility of the Joint Chiefs of Staff to review the major material and personnel requirements of the military forces in accordance with strategic and logistic plans. In carrying out this responsibility, the Joint Chiefs of Staff analyze and consider the logistic implications of war plans while they are being developed. In addition to implication tests, industrial feasibility tests are conducted on strategic plans to determine whether or not manpower and industrial capacities are adequate to support the plan and to determine the status of the logistic readiness of the Armed Forces. The results of such tests indicate deficiencies that may exist and provide a back-drop from which to proceed in developing the budgetary plans required to fill in the gaps. Or if such correction of deficiencies is not practicable, the test will provide the Joint Chiefs of Staff with the basis for making such modifications to the strategic plan as deemed advisable.

III. INTEGRATION OF CURRENT STRATEGIC AND LOGISTIC PLANNING.

A. Types of Joint Plans.

1. The Joint Chiefs of Staff are responsible for the development of strategic and logistic planning in accordance with the National Security Act of 1947 as amended in 1949. The Joint Chiefs of Staff Program for Planning calls for three joint strategic plans: The Joint Long Range Strategic Estimate, The Joint Strategic Objectives Plan and The Joint Strategic Capabilities Plan. Each strategic plan has an appropriate logistic plan to support it. The logistic plan may be in the form of a separate plan or may be an annex to the strategic plan.

2. The Joint Long Range Strategic Estimate is based on long term national objectives and takes into consideration the predicted strength and capabilities of probable enemies, the development of new weapons, progress in technology, improved production techniques and the predicted status of national resources. The Estimate covers the period between 5 and ten years in the future. The logistic guidance for the Estimate is developed concurrently and is in the form of a logistic annex. It provides guidance for continuity of logistic support required as a result of technological advances.

3. The Joint Strategic Objectives Plan covers a four year buildup based on mobilization objectives. It must be disseminated well in advance of its D-day to permit the preparation of budgets by the services, the attainment of peacetime levels of readiness through the expenditure of appropriated funds during the year preceding D-day and the careful determination of total mobilization requirements. The period between the date of approval and the D-day of the plan must be 3 years if the purpose of the plan is to be realized. The Objectives Plan reflects more reliable intelligence than the Joint Long Range Strategic Estimate and is based on requirements and restrictions imposed by time and material on the development of military strength. The logistic guidance for the Objectives Plan is also in the form of an annex. This annex indicates the logistic action to be taken to provide for peacetime readiness and for mobilization.

4. The Joint Strategic Capabilities Plan is, in essence, an emergency war plan based upon current military strength and the feasible buildup during a 4 year period subsequent to an emergency. It is based upon the best current intelligence. It is limited in its basic objectives

to those which are within the immediate capabilities of the military forces in being and that of industry. The Capabilities Plan is developed in detail, indicating phased operations, since it must provide guidance for detailed service plans and the supporting emergency plans of unified commanders and certain other commanders. The logistic guidance for the Capabilities Plan is a separate logistic plan. The logistic plan supports the phased operations of the strategic plan and points out the pre-D-day measures to be taken to fill existing deficiencies.

B. Joint Chiefs of Staff Program for Planning.

1. In 1949, the Joint Chiefs of Staff directed the Joint Staff to develop a coordinated program for planning. The purpose of the program was to provide coordination and integration of war plan and planning. Previous to this time, the Joint Chiefs of Staff war plans consisted of a Joint Outline Emergency War Plan which was revised annually and a Joint Mid-Term War Plan. A Joint Long Range War Plan had been developed by the Joint Staff but only the Estimate contained therein had been approved for planning purposes. These plans had not been closely related and had precipitated crash planning from time to time.[9]

2. The Joint Chiefs of Staff Program for Planning was approved in July 1952. The program schedules planning cycles for the development of the 3 strategic plans, their logistic plans or annexes and other associated planning such as budgetary planning. It provides a time schedule for the preparation of the plans and the Joint Chiefs of Staff action thereon, as well as the coordination and integration of the three plans in the planning process.

3. The time schedule for the preparation of the three plans and the action to be taken by the Joint Chiefs of Staff, as shown in Appendix I, covers one calendar year. Two months are allowed for the preparation of the intelligence estimate for each plan, although these are prepared at three different times of the year. Four months are allowed for the preparation of each of the plans and for the review and approval of the Joint Intelligence, Strategic and Logistic Plans Committees. An additional two months are allowed for the consideration, approval and dissemination by the Joint Chiefs of Staff.

4. The Logistics Plan to support the Joint Strategic Capabilities Plan is scheduled to be completed within 30 days after approval of the Capabilities Plan by the Joint Chiefs of Staff. Since the supporting logistic plans for the Joint Long Range Strategic Estimate and the Joint Strategic Objectives Plan are annexes to the plans, they are developed and approved concurrently with the plans themselves.

5. The interrelationship between the Joint Capabilities Plan and the Joint Objectives Plan may be seen on the chart contained in Appendix II. After the Objectives Plan has been approved and disseminated by the Joint Chiefs of Staff, the services commence budget planning cycles based upon the level of peacetime readiness to be attained during the year immediately preceding the D-day of the plan. The preparation and support of the budget and the action required by Congress consumes two years. In the meantime, the services are developing supporting plans from which total mobilization requirements can be developed. The Joint Staff subjects the plan to an industrial feasibility test for the consideration and review of the Joint Chiefs of Staff.

6. During the third year of the planning cycle of the Objectives Plan, the funds appropriated as a result of the two year budget planning cycle are used to attain the best possible level of peacetime readiness. It will be noted that the D-day of the Objectives Plan coincides

[9]Berrigan, Paul D., Col. USA., Development and Testing of Strategic Plans by the Joint Chiefs of Staff. Wash., D.C. ICAF, 18 Nov 1952. p. 5. (L53-55)

with the D-day of the Capabilities Plan. It can be seen that the level of readiness attained by the D-day of the Objectives Plan dictates the scope of the related Capabilities Plan both with regard to forces in being and to mobilization capabilities. The estimated capabilities to be incorporated in the revised Capabilities Plan is determined by the Joint Chiefs of Staff through the review of the industrial feasibility test of the Objectives Plan.

7. Further interrelationship of plans and estimates is depicted on the chart contained in Appendix III. This chart shows the time schedule for the annual revision of each of the three strategic plans. It illustrates how tangible gains in capabilities acquired in the first Objectives Plan should be reflected in the second revision of the Capabilities Plan. The long lead time requirements of the first year of the Long Range Strategic Estimate will be considered in the second year of the Objectives Plan and will be reflected in the spending year thereof. Thus it can be seen that the family of plans, as developed and scheduled by the Joint Chiefs of Staff Program for Planning, is a family with interdependent relationships.[10]

IV. THE DEVELOPMENT OF REQUIREMENTS.

A. Purpose.

1. The determination of requirements is the first step in the formulation of any logistic plan and it is also inextricably involved with strategy and tactics.[11] Some of the many purposes for which requirements are computed are: to determine budget estimates, civilian and foreign aid requirements, stockpile objectives and the testing of military plans. At the highest level of planning, the development of requirements serves, among others, the important function of facilitating the testing of strategic plans for logistic feasibility.

2. Strategic Plans developed by the Joint Chiefs of Staff are broad outline war plans. These plans indicate the general concept of operation and outline the time phased tasks to be undertaken together with a statement of the major tactical units required to perform these tasks.[12] Since the security of the nation and our allies is dependent upon the soundness and feasibility of these plans, it is encumbent upon the Joint Chiefs of Staff to test strategic plans and to make such revisions as may be indicated by the tests. The tests must be comprehensive and must include the testing of the capability of the nation and its economy to make available the manpower, materials and production facilities necessary to support a military strategy on the desired scale or on a scale of operations that will insure ultimate victory.

3. To test a strategic plan, it is necessary to develop the military requirements essential to the support of the time phased operations as outlined in the war plan. These requirements include such items as personnel, munitions, weapons, equipment, transportation, training facilities and supplies. In addition, requirements of the civilian supporting components and the civilian economy itself must be taken into consideration. Furthermore, those military and civilian requirements of our allies that exceed their own capabilities must be included along with our own, if we are to plan soundly. Only by knowing the full extent of the requirements to fight a war or to implement a strategic plan, can it be determined whether or not the overall strategy of the plan can be supported by the national economy.

---

[10] Ibid. p. 15.

[11] Eccles, Henry E., Capt, USN, "The Determination of Requirements", Operational Naval Logistics, Wash., D.C., Bureau of Naval Personnel, Apr 1950, p. 14.

[12] Industrial College of the Armed Forces, Feasibility Tests and Checks of Material Requirements. Wash., D.C. 21 Dec 1949. p. 3. (SR50-13)

B. Methods.

1. There are several distinct types of requirements estimates.[13] Each serves a particular purpose and should not be confused, one with the other. The definition of requirements from the standpoint of the military forces is the need or demand for personnel, equipment and supplies, resources, facilities or services, by specific quantities, for specific periods of time or at specified times.[14] Production requirements, on the other hand, are simply the needs of the claimant agencies for the production of industry to fulfill their wartime missions.[15] To the Office of Defense Mobilization, requirements mean the requirements of the civilian economy and the military support areas as well as those of the military and the requirements to fill the gaps or deficiencies in a balanced mobilization base. However, for the purposes of this paper, only those types and methods of developing requirements which are of primary interest to the services need be considered.

2. Requirements based on Joint Chiefs of Staff strategic plans.

a. Joint Chiefs of Staff strategic plans, sometimes referred to as "joint outline war plans", are statements of national objectives, an estimate of enemy capabilities, the concept of operations, the tasks to be performed and the time-phased employment of major tactical units. The strategic plan is not, in any sense, an operation plan. When approved and disseminated, it provides guidance to the services for the development of service supporting plans. From the operational pattern of the Joint Strategic Plan, the services develop their operational and supporting logistic plans which become the basis for determining service mobilization requirements. Further guidance is furnished by the Joint Chiefs of Staff through the "Joint Logistics Policy and Guidance" paper. This paper is a compilation of the Joint Chiefs of Staff approved logistic policies and indicates the assignment of responsibilities for such functions as sea and land transportation, harbor defense, procurement of food and other supplies and construction. The Joint Logistics Policy and Guidance is applicable to any strategic plan or estimate and, subject to periodical revision and modification by the Joint Chiefs of Staff, it provides the services constant or static guidance for all types of logistic planning and requirements development.

b. The logistic plan or annex supporting the Joint Chiefs of Staff strategic plan, although not detailed in content, sets forth the specific guidance not covered in the Joint Logistics Policy and Guidance and the guidance upon which to develop service logistic plans and requirements. For instance, the level of reserve stocks, resupply levels and the policy with regard to maintenance and overhaul of equipment will be prescribed. It insures maximum use of the available logistic structure and insures that the logistic pattern best supports the operational pattern, taking into consideration the area of planned operations and such details as the tempo of operations, timing of operations and climatic conditions, to mention a few.

c. The detailed computation of requirements are developed by each service in accordance with the guide lines provided by the Joint Chiefs of Staff, as previously indicated. But this broad policy guidance must be supplemented by service policy. In the Navy, the guidance for the detailed determination of end item requirements is stated by the Chief of Naval Operations. The detailed computations of hundreds of thousands of end items of equipment and supplies is accomplished by the Technical Bureaus of the Navy Department, and are based on the supporting plans

[13] Lawrence, William N., Role of ODM in the Determination of Requirements. Wash., D. C., ICAF, 17 Dec 1953. p. 7.

[14] Joint Chiefs of Staff, Dictionary of United States Military Terms, Wash., D. C., June 1950. p. 75.

[15] U.S. Munitions Board, Production Allocation Manual, Wash., D. C., GPO, Mar 1950, p. 45.

of the Fleet Commanders, District Commandants, Area Commanders and other shore activities as well as on the service plan itself. The end item requirements so computed are reviewed for validity by the CNO. From these requirements the Bureaus determine materials requirements which are submitted to the Office of the Chief of Navy Material for review and consolidation.

d. The Army and Air Force follow the same general procedure as the Navy in developing requirements. Variations in administrative procedures result from differences in organization. In the Air Force, for instance, the Air Material Command and the technical services which are physically located outside the Washington, D.C. area, perform functions similar to that of the technical bureaus of the Navy, and in Air Force Headquarters, the Office of the Deputy Chief of Staff for Material, is, generally speaking, the counterpart of the Office of Naval Material.

e. Planning factors play a major role in the determination of service requirements. There are three types of factors currently in use.

(1) Joint agreed factors - those applicable to one or more of the three services and which are acceptable to all, such as personnel factors.

(2) Joint factors - those applicable to all three services such as the amount of lubricating oil used per barrel of gasoline.

(3) Unilateral factors - those applicable only to one service with no requirements for the other two, such as the Army division slice.

f. The development of requirements for the purpose of testing the feasibility of a Joint Chiefs of Staff strategic war plan provides for programs to stockpile strategic raw materials, policies with regard to levels of war reserves and reserve production facilities and equipment, and guidance for budget estimates and requests during pre-D-day years.

g. Requirements are computed for each new joint strategic plan and when a current plan has been materially modified. Each new supporting logistic plan calls for a different set of requirements to meet a new situation. Logistics plans must be translated into requirements for hundreds of thousands of items of supplies and equipment in terms of specific time and quantities.[16]

h. Each service consolidates the requirements as developed by its technical bureaus or services and submits total requirements to the Assistant Secretary of Defense for Supply and Logistics. The time phased requirements so submitted include a list of hardgoods end items such as tanks, ammunition, aircraft and vehicles and a compilation of semi-finished and raw materials required to produce them. The materials requirements include only those categories such as aluminum, steel and copper which are of a critical nature in a mobilization. From time to time, the Office of the Secretary of Defense for Supply and Logistics may call for the submission of other lists of requirements to be tested such as construction, POL, transportation, personnel or supplies, depending upon the extent of modification the particular strategic plan may have undergone. But this does not mean that the services do not compute total requirements in each case. Upon receipt of the total military requirements, the Office of the Assistant Secretary of Defense for Supply and Logistics conducts an industrial feasibility test balancing total requirements against the available resources of the nation.

3. Requirements based on unilateral service plans.

a. The determination of requirements for unilateral service plans is not considered to be desirable or a satisfactory procedure. In this type of planning, there is always a

[16]Industrial College of the Armed Forces, Economic Mobilization Studies - Requirements, Wash., D. C., Oct 1952. p. 18. (R160)

possibility that more than one service will develop requirements for the same task or services and that each may base plans on the use of the same facilities. When planning is not coordinated and based on the same guidance, there is a possibility of duplication and overlapping. The result will be unreasonably high total requirements. In war, such planning may well prove to be detrimental to the conduct of operations as well as being wasteful of resources and production facilities. However, when guidance from the Joint Chiefs of Staff or higher authority is lacking or is delayed, it is necessary for the services to proceed with planning along unilaterally developed guide lines. The most common example of unilateral planning resulting from lack of timely guidance is the annual budget estimate. Also, the services have found it necessary, on occasions, to proceed with mobilization planning based on unilateral plans. It is the stated purpose of the Joint Chiefs of Staff Program for Planning to cure this situation in the future. Unforeseen emergencies, such as the Korean emergency, which require a limited or special type mobilization, will not always fit currently approved joint mobilization plans and may call for a certain amount of unilateral planning in the development of requirements for budget and other planning purposes.

b. In the meantime, the services develop requirements for annual budget requests for funds to cover current peacetime commitments and the maintenance and training of forces in being as well as to improve the readiness to mobilize. These requirements are recomputed in considerable detail for procurement purposes, after funds have been appropriated by the Congress.

4. Requirements based on estimated industrial capacity.

This type of requirements are computed by each service based on an estimate of the capability of industry to produce the end items it requires. The estimate, in each instance, is based on accumulated capacity data concerning the suppliers with whom the individual service agencies are doing business and those with whom mobilization production planning has been accomplished.[17] A computation of requirements on this basis is theoretically feasible from an industrial production standpoint but it is not necessarily feasible from the standpoint of the availability of critical materials or supporting resources and components.

5. Requirements based on the Gross National Product Method.

a. The Gross National Product method of determining requirements is also known as the "Mobilization Production Planning Program." The concept of this method is to determine the maximum portion of an estimated GNP that can be allocated to the military while still supporting a minimum civilian economy and providing essential exports to our allies and then to base the strategic plan on the maximum striking force that can be obtained with the military slice of the GNP pie.

b. This method was adopted by the Joint Chiefs of Staff in December 1952. ODM also accepted the plan as a method of testing the capability of the total economy to support a mobilization. The GNP must be estimated for an assumed M-day in the future and for the three subsequent years during which the national economy would be converted to a war economy at a maximum rate. In determining the GNP and the maximum feasible expansion of industrial capacity, the following basic assumptions were agreed upon by the Joint Chiefs of Staff and ODM:[18]

---

[17] Lawrence, Op. Cit. p. 8.

[18] Matejka, Jerry V., Maj Gen, USA. Determination of Requirements to Support a Full Mobilization Production Program. Wash., D. C., ICAF, 25 May 1950. p. 6. (L50-140)

(1) The assumed M-day.
(2) Losses from sabatoge or bombing.
(3) Minimum civilian standard of living.
(4) The rate of annual increase of the GNP.
(5) The availability of critical materials.
(6) The types of end item requirements to be computed and submitted by the services.
(7) The basic value of the dollar to be used in reducing the value of the GNP to a common level.

c. ODM estimates the GNP and determines the sizes of the slices of the pie. The GNP is determined by multiplying the total civilian labor force by the length of the work week and the productivity of the worker. The total estimated production, reduced to a fixed dollar value, is divided to provide the maximum to the military, the minimum for the civilian economy and the essential exports to support our allies. The Office of the Assistant Secretary of Defense for Supply and Logistics divides the military slice, for each of three years, among nine categories of hardgoods, using the estimated industrial capacity of the nation and war experience factors as guides in determining the proportions. The nine categories are: aircraft, weapons, munitions, guided missiles, ships, combat vehicles, noncombat vehicles, electronics and other hardgoods.

d. Within these category ceilings, the Joint Chiefs of Staff allocate the hardgoods to the services by dollar values by programs. It is then the prerogative of the services to determine what they wish to buy under the given dollar ceiling. The end items in each category become the requirements under the GNP Method and are based on needs of each service to carry out missions assigned in the strategic plan. In addition to end items, for each of three years of mobilization, the services submit the names of current and potential producers, the dollar value of end items and the requirements for critical materials such as copper, aluminum and steel.

e. Civilian agencies plan production programs for transportation, petroleum, agriculture, energy, housing, materials and export. For each of these programs, requirements are determined for each of the major resources: manpower, materials, production equipment and facilities.[19] These requirements, along with the military requirements are submitted to ODM for consolidation.

f. ODM then determines the availability of national resources and balances this against the total military-civilian requirements. Since the program is feasible, dollar wise, to begin with, it is only necessary to check the balance of resources. For instance, it is necessary to determine whether or not there is enough energy to meet the total production scheduled or potential engine production balances with potential aircraft production.

g. This test will uncover deficits and surpluses in national resources. Some deficits can be met by redistributing resources or through expected growth of availabilities. It is the responsibility of ODM to recommend to the Administration and to the Congress the action to be taken to eliminate deficiencies. Recommended action may include provisions for stockpiling critical materials and machine tools, the expansion of energy, programs for technical training of personnel or the level of war reserves to be maintained. If it is determined that the nation is able and willing to close the gaps, in the time allowed, the military plan is

[19]Lawrence, Op. Cit. p. 9.

feasible. If, on the other hand, it is decided that the nation is unable or unwilling, the military plan is infeasible in certain known respects and adjustments must be made in the plan.

C. Evaluation of Methods.

1. In planning for the security of the nation, it is essential that strategic war plans be reasonably feasible. To plan strategy without investigating logistic capability is to court disaster. Therefore, it follows that in testing the capability of the nation to carry out an approved strategy, only those requirements which are developed from the time phased operational pattern of that strategy would be acceptable. Furthermore, since another global war will strain the resources and economy of the nation to its limits, it is essential that the planning accomplished by the individual services be closely coordinated in the interest of economy of resources and of achieving maximum results from those which are available. Consequently, only those military requirements which are developed from approved Joint Chiefs of Staff strategic plans should be used for purposes of determining the required levels of war reserves, stockpiling programs and the deficiencies on the industrial mobilization base.

2. Military requirements, however, are only one part of total requirements for mobilization. It is also necessary to know the requirements of the segments of the civilian economy which support the military effort and the requirements of the civilian economy itself. Therefore, the GNP Method of developing requirements serves a useful purpose and an essential function in pointing up the civilian requirements and in measuring total available resources. This information can be of great assistance to the Joint Chiefs of Staff in subsequent strategic and logi stic planning. Furthermore, in pointing up deficiencies and imbalance in our mobilization base, it provides a basis for remedial legislative action.

3. The GNP Method does not, necessarily, present the requirements to win a war. Only requirements based on the complete guidance of a Joint Chiefs of Staff strategic plan can be considered to be sound for the purposes of determining the pre-D-day policies and programs required to insure ultimate victory.

SUMMARY

1. Prior to World War II, warfare had been confined to one major theater of combat operations. The logistic support of strategic plans raised few complex problems in that this support could be directed towards a single objective and needed to take into account but a minimum of alternate strategies. Furthermore, war itself was less complex because of the nature of weapons and the relative immobility of forces. In World War II we learned to support combat operations in as many as eleven major theaters and under as many varieties of geographical and climatic conditions. The learning, however, was at the expense of time and the effective use of forces in being. It was learned that the successful execution of grand strategy was dependent upon the capability of the nation's resources, industrial potential, manpower and services to support it. The total potential of the nation to support a war had never been required or fully tested. It was something of a shock to find that the resources of the nation were not unlimited.

2. With respect to military planning, probably the major lesson learned from World War II was the need for concurrent logistic and strategic planning. It was evident to the Joint Chiefs of Staff early in World War II that a logistics planning staff that determined the feasibility of strategic plans was vital to the planning effort and that the findings of the logistic planners dictate the phasing of combat operations. Since time itself is of the essence to the logistic

planner, it was soon realized that the logistic planner must follow closely the development of strategic plans. The logistic planner must be ever cognizant of tentative strategic plans in order to gain an idea of the area, the scope, the tempo and the time phasing of planned operations. With this information the logistician can determine extraordinary requirements and thus uncover possible bottlenecks in procurement or shortages in resources. When war is in progress, the logistic planner can test plans rapidly by testing the availability of known critical items against the requirements of planned operations. The availability of shipping alone dictated the phasing of many of the operations scheduled in the various theaters of World War II. But in the early phases of a war, while industry is mobilizing, it is of the utmost importance that the logistic planner be familiar with the broad strategy in order that the expansion of industry may be properly and effectively scheduled.

3. Concurrent planning developed from the need of close team work between the logistic and strategic planning at all planning levels during World War II, in order to expedite the determination of requirements. Although considerable emphasis has been placed on concurrent planning, in the meantime, unilateral planning has not entirely disappeared because timely guidance from higher authority has not always been available and because Joint Chiefs of Staff strategic plans have not been completely integrated. This has resulted, at times, in duplication and overlapping in planning and in the determination of requirements. To correct this obvious deficiency, the Joint Chiefs of Staff developed in 1952 a "Program for Planning" which sets forth a time schedule for the development of strategic plans and establishes a logical interdependency between long range, mid-term and short range plans.

4. The Joint Chiefs of Staff Program for Planning also sets forth the purpose of each of the three strategic plans and the manner in which they are to be tested for logistical implications and industrial feasibility. Thus the mid-term or "Objectives Plan" provides budgetary guidance and a basis for expanding the mobilization base and determining the required level of war reserves. The level of readiness attained through the expenditure of appropriated funds obtained through the guidance provided by this plan determines the level of capabilities for the time related "Capabilities Plan." This plan also provides a basis for determining the military requirements used in testing the industrial feasibility of the strategy developed by the Joint Chiefs of Staff.

5. Although military requirements can only be determined from the guidance of a Joint Chiefs of Staff strategic plan, military requirements alone do not represent total mobilization requirements. It is necessary to know the requirements of the military supporting segments of the civilian economy, the requirements of the civilian economy itself and the estimated needs of our allies. The Gross National Product Method of determining requirements currently being employed by the Office of Defense Mobilization provides a means of determining non-military requirements and the deficiencies in the mobilization base. Since it places a ceiling on military expenditures during the first three years of a war, it does not represent a realistic approach to the determination of military requirements. But the fact that the Office of Defense Mobilization has been established and is actively engaged in the determination and testing of overall requirements for war attests to the awareness of the interrelation and the interdependency of logistic and strategic planning both in the Executive and Legislative branches of the government.

CONCLUSIONS

It is concluded that:

1. One of the most important lessons learned from World War II is the principle that in a major military command there is an essential need for a logistics staff agency that determines the feasibility of the operational plans of that command and that a knowledge of the principles of logistics is a necessary qualification for the command of military forces.

2. Concurrent planning has evolved from the demonstrated need during World War II of closer team work between the logistic and strategic planners in all planning echelons in order to facilitate and expedite the determination of requirements.

3. Jointly agreed guidance emanating from the Joint Chiefs of Staff for the determination of requirements is essential in order to avoid duplication and overlapping in service planning and thus avoid unrealistically high and unnecessary demands on the national economy.

4. The National Security Act of 1947 and the Joint Chiefs of Staff Program for Planning foster the principles of concurrent planning, the integration of strategic and logistic plans and the elimination of duplication in the development of requirements.

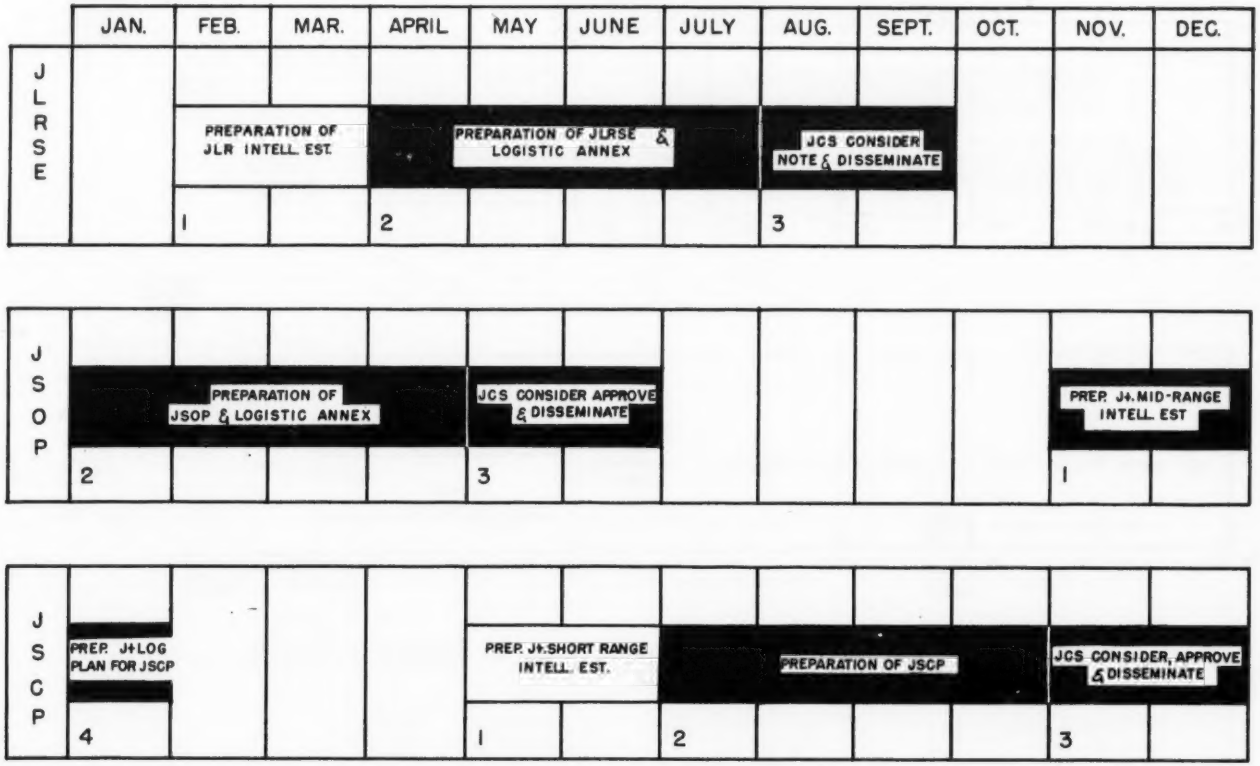
5. True military requirements can stem only from the complete guidance of approved Joint Chiefs of Staff strategic plans but to test the capability of the national economy to support full mobilization, the requirements of the civilian economy, the military support areas and our allies must be determined and be taken into consideration.

6. The Gross National Product Method of determining requirements can be of great value in future mobilization planning because it will present, for the first time, the total resources available to support a mobilization and the shape and balance of the mobilization base. It will compute for the first time, total non-military requirements which, when added to military requirements, will provide a realistic basis for testing the Joint Strategic Objectives Plan for logistic feasibility. Such a test, when completed, will be a backdrop for the use of the Joint Chiefs of Staff for revising the Joint Strategic Capabilities Plan.

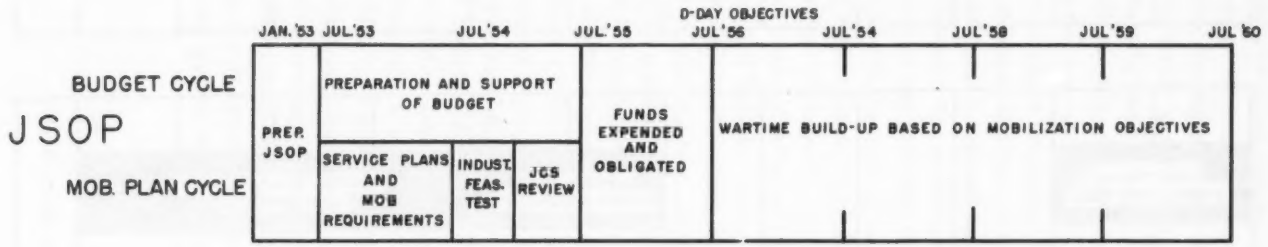
7. There is an awareness of the interrelation and interdependence of logistic and strategic planning both at the highest planning levels and in lower echelons. That this awareness exists is evidenced by legislative action such as the National Security Act wherein the Joint Chiefs of Staff are assigned logistic as well as strategic planning responsibilities and the organization of the Joint Staff to include a Joint Logistics Plans Group. This awareness is further evidenced by the reorganization of the Executive Branch of the government including an Office of Defense Mobilization to coordinate mobilization planning.

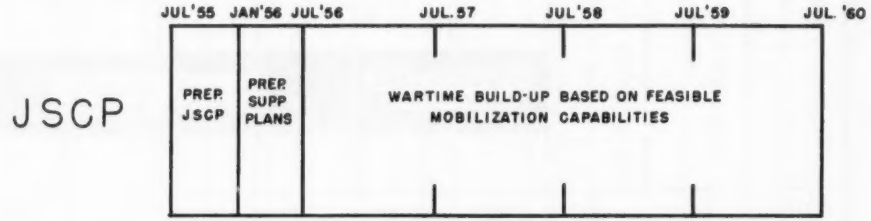
8. The Joint Chiefs of Staff, through the organization of the Joint Staff and jointly agreed directives such as the Program for Planning, have laid the groundwork and provided adequate means to integrate logistic and strategic planning.

APPENDIX I
Annual Schedule for Strategic Planning



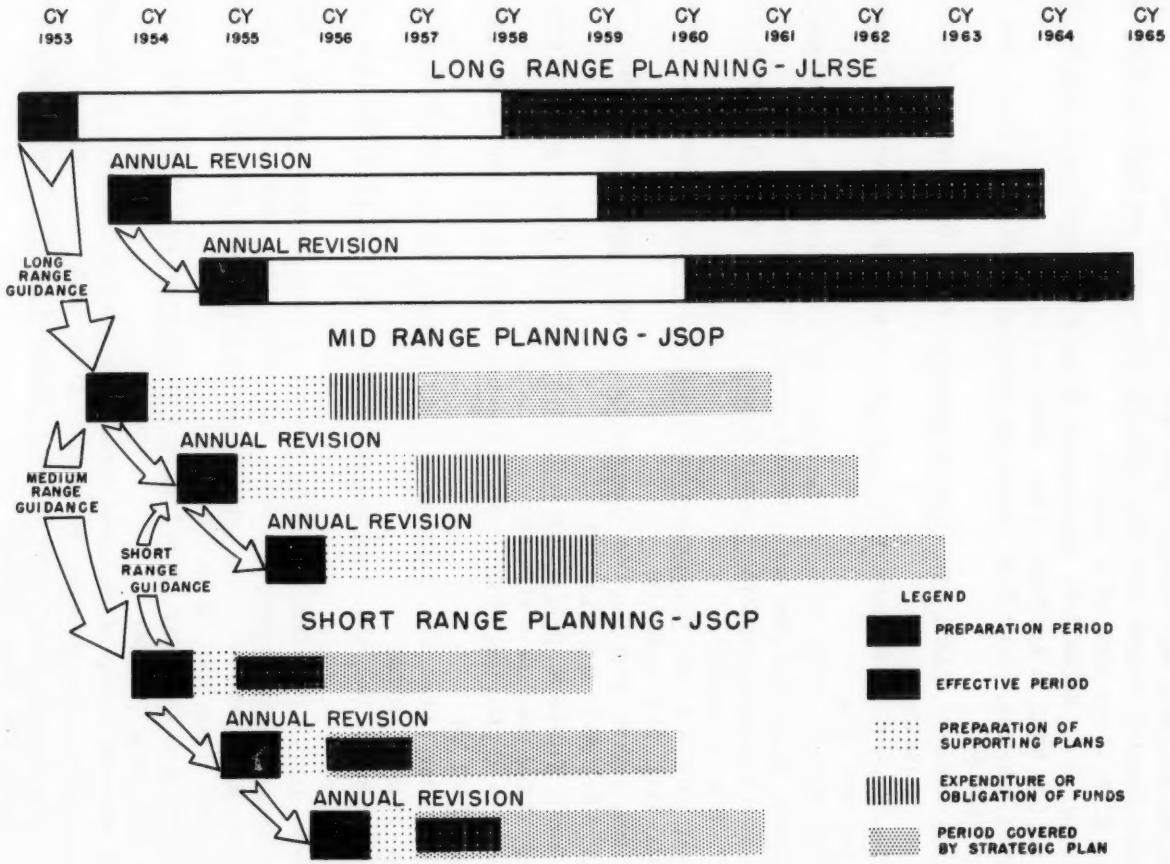
APPENDIX II
Relationship between Budget and Mobilization Aspects of the JSOP and its Related JSCP





APPENDIX III
Interrelationship of Plans and Estimates = Planning Cycle

APPENDIX III
Interrelationship of Plans and Estimates = Planning Cycle



BIBLIOGRAPHY

[1] Badger, Oscar C., VADM, USN. The Influence of Logistics on Military Strategy. Newport, R.I., Naval War College, 11 Dec 1952. 12 p. (U 168 B2).

[2] Badger, Oscar C., VADM, USN. The Problems of Command and Logistics Work. Wash., D.C., ICAF, 18 Nov 1949. 14 p. (L50-53)

[3] Campbell, R. L., RADM, USN. Development of Logistic Implications of Joint War Plans. Wash., D.C. NWC, 13 Apr 1953. 39 lvs. (NWC-53 I C151)

[4] Carney, Robert B., VADM, USN. Concurrent Planning. Wash., D.C., ICAF, 6 Dec 1948. 10 p. (L49-56)

[5] Carney, Robert B., VADM, USN. Logistic Planning. Wash., D.C., NWC, 25 Feb 1948. 18 lvs. (U 168 C2)

[6] Eccles, Henry E., CAPT, USN. Basic Logistics. Newport, R.I., Naval War College, 30 Aug 1950. 23 p.

[7] Eccles, Henry E., CAPT, USN. The Determination of Requirements. Operational Naval Logistics. Wash., D.C., Bureau of Naval Personnel, 1950. 218 p.

[8] Esposito, Vincent J., Col, USA. Logistic Planning for War. Wash., D.C., NWC, 4 Feb 1947. 23 p. (U 168 E8)

[9] Elliott, William Y., Dr. Organization for Civilian Requirements. Wash., D.C., ICAF, 14 May 1948. 24 p. (L48-140)

[10] Hinrichs, John H., Brig Gen, USA, Computation of Military Requirements. Wash., D.C., ICAF, 9 Dec 1953. 21 p. (L54-64)

[11] Lawrence, William N., Role of ODM in the Determination of Requirements. Wash., D.C., ICAF, 17 Dec 1953. 19 p. (L54-69)

[12] Lindsay, R. C., Maj Gen, USAF, Strategic Planning. Wash., D.C., NWC, 1 Mar 1951. 19 p. (U 162 L5)

[13] Lutes, LeRoy, Lt Gen, USA. Logistics in Grand Strategy. Wash., D.C., NWC, 13 Feb 1950. 13 p. (U 168 L8)

[14] Lutes, LeRoy, Lt Gen, USA. Munitions Board. Wash., D.C., ICAF, 27 Oct 1948. 20 p. (L49-33)

[15] Matejka, Jerry V., Maj Gen, USA. Determination of Requirements to Support a Full Mobilization Production Program. Wash., D.C., ICAF, 2 Dec 1952. 30 lvs. (L53-63)

[16] Pierson, Albert, Maj Gen, USA. Logistic Planning Activities of the Joint Staff. Wash., D.C., NWC, 6 Mar 1951. 29 lvs. (U 162 L5)

[17] Simenson, E. G. Col, USAF. Joint Logistic Planning by the JCS. Wash., D.C., ICAF, 21 Nov 1951. 26 p. (L52-56)

[18] U. S. Industrial College of the Armed Forces. The Effect of Concurrent Planning on the Determination of Requirements. Wash., D.C., 21 Dec 1948. 65 p. (SR49-6)

[19] -- -- The Elements Which Affect the Determination of Material Requirements. Wash., D.C., 19 Jan 1951. 53 p. (SR51-9)

[20] -- -- Feasibility Tests and Checks of Material Requirements. Wash., D.C., 21 Dec 1949. 43 p. (SR50-13)

[21] -- -- Economic Mobilization Studies-Requirements. Wash., D. C., Oct 1952. 44 p. (R160)

[22] -- -- Joint Chiefs of Staff. Program for Planning. Wash., D.C., 14 July 1952. v. p.

[23] -- -- Dictionary of United States Military Terms. Wash., D.C., June 1950.

[24] -- -- Munitions Board. Production Allocation Manual. Wash., D.C., GPO, Mar 1950.

[25] -- Law. National Security Act Amendment of 1949. Approved 10 Aug 1949. 16 p. (Public Law 216, 81st Congress)

* * *

MOBILE LOGISTICS SUPPORT IN THE "PASSAGE TO FREEDOM" OPERATION

Commander Randolph Meade, Jr.
Supply Corps, U. S. Navy

and

Lt. Commander Carl A. Fischer
Supply Corps, U. S. Navy

Not a soul involved in the "Passage to Freedom" operation in French Indo-China heard the statement, "This is a drill." Feeling that the operation of an actual support mission, not an exercise, will most graphically depict the requirements and execution of modern mobile logistic support, it is the writer's intention to describe the support of the Amphibious Forces by Service Squadron THREE in the evacuation of refugees, fleeing the red arm of Communism, from northern to southern Viet Nam. It will be pointed out that many lessons were learned and shortcomings were encountered. It will be well to note those work simplifications developed or utilized which helped Service Squadron THREE to expedite the job or to ease its burden.

Ever since the siege of Dien Bien Phu, and probably earlier, the U.S. Forces had been faced with the eventuality of an evacuation under duress. This could have been caused by the collapse of the French, the wholesale defection to the Communists by previously loyal Vietnamese, or the launching of overwhelming forces by the Communist forces from China. This would have forced an evacuation of French and loyal forces and their equipment under conditions of war and with a requirement for all haste in its execution.

Accordingly, there were in existence a number of Operating Plans written toward this situation, and of course, Rear Admiral Roy A. Gano's Service Squadron THREE was the designated logistics support agency for each. In recognition of the variety of plans, the requirement that Logistic Support force be able to respond to any and all of them, and the particularly explosive situation to be faced in Indo-China, Commander Service Squadron THREE's Staff had been in the process of rewriting and consolidating its own Op Plans. It was apparent that actually one good plan, written in sufficiently broad terms, would encompass all the operational situations covered by the many plans. It is perfectly clear that a service squadron does the same thing in peace or war, the difference being only in amounts delivered and the conditions of security surrounding support operations.

Fortunately, Service Squadron THREE had just finished, on 8 August 1954, a new Op Plan covering all the other plans and had a good reference point. Additionally, there had been considerable, not altogether academic, speculation on what would be done if the Navy precipitated into an Indo-China situation that had gotten out of hand. Hence, we were not wholly unprepared when Admiral Sabin's Amphibious Forces in WESTPAC were ordered to assist the French in their evacuation operations.

Task Force 90, the Amphibious Force, had had some prior warning from a high level, highly classified, exchange of messages, in which concern was expressed over the slowness and inadequacy of the evacuation processes, an evacuation of the Hanoy-Haiphong area, permitted by the armistice agreement. It was estimated that a nebulous number of people, somewhere between 100,000 and one million, would apply for evacuation, and knowledge of the rate

of their application, their haste, their physical condition and facilities for the evacuation were all equally nebulous for planning purposes. Beach sites had been picked for evacuation under duress, so that much at least was regarded as fairly firm.

Commander Task Force 90's call for help caught Commander Service Squadron THREE just going out to sea. It was the time for the staff's periodic rotation from Sasebo to Yokosuka. This was fortunate in that all ports normally covered by inport replenishment groups were covered by splinter staffs, leaving the main body of the staff free to plan. Nevertheless, it was untimely in that it offered little chance for direct or consolidated conference with CTF 90 on his planning as his sailing date was prior to the arrival of COMSERVRON THREE at Yokosuka. Fortunately the Staff Supply Officer of COMSERVRON THREE was only that day returning to Yokosuka from a trip to the Philippines, and was able to visit CTF 90 staff, obtain a briefing on that staff's thinking and planning, read the background dispatches, and most important of all, obtain two copies of CTF 90's Op Plan covering the situation as they came off the presses. He was also able to assist in assigning overriding supply priorities to ships being readied for the operation.

In the meantime CTF 90 had told CTF 92 (COMSERVRON THREE) by message what was believed to be the requirements for the operation in the way of a replenishment group. After a quick look at his forces, CTF 92 responded with a "Can do - with the exception of the water carrier requested, which is not available." It was by way of a repetition of Joe Taussig's famous "We are ready now!" at Queenstown. Incidentally, the enthusiastic backing always received from Commander Service Force, U.S. Pacific Fleet, is best exemplified by his comment on Admiral Gano's message. It was simply one word — "ATTABOY."

In addition to the entirely ordinary requirements of provisions and fuel for the operating and support forces were the exotic requirements for extra-ordinary quantities of fish and rice, for chop sticks, for rice sleeping mats, for buckets for sanitary purposes, for life jackets and for many epidemic control medicines and equipments. These could require specific additional lift to the area. Plans had to encompass provisions for the three "M's" of morale, mail, and aMusement. Money would be a problem, for the operational area was not in the MPC zone, and hospital services would be essential if intimate contact were made with any of the diseases present in the area in epidemic proportions.

Immediately upon his arrival at Yokosuka, COMSERVRON THREE and members of his staff met with the staff of COMNAVFE in a joint conference at which additional background information was provided. It was apparent that SERVRON THREE was qualified and prepared to do the job requested, with only two exceptions, namely, lack of sufficient water making or water carrying capacity, and an immediate source of currency guidance. These two items, one physical, one informational, NAVFE staff undertook to get. The ball was then tossed to COMSERVRON THREE by formally designating him as the logistic agent for CTF 90.

At this point it might be well to quote the Chief of Naval Operation's press release on the subject of the operation contemplated.

> "At the request of the Vietnamese and the French Government, the Navy is assisting in the evacuation of refugees from North Viet Nam (Haiphong area) to South Viet Nam (Saigon area). The U.S. assistance in the sea evacuation will be under the overall command of Admiral F. B. Stump, Commander in Chief, Pacific and US Pacific Fleet, with Admiral Sabin, Commander of Amphibious Forces, WESTPAC, in direct charge of the sea operations. The number of persons involved is not known. The original estimate was 100,000. However,

both the French and the Vietnamese officials on the scene indicate that a wide discrepancy exists (250,000 to 800,000) regarding the total number of refugees to be evacuated. This is due principally to the failure of many of the evacuees to register until assured of evacuation for fear of reprisal against their families. Regardless of number, it is considered desirable that the U.S. provide maximum humanitarian assistance and that we take full military and psychological advantage derived therefrom."

Immediately work began constituting TG 92.3, the objective area mobile support group. It was decided that one oiler, one small reefer, and a small AK with dry provisions should go; a Light Cargo Ship (AKL) would be deployed to lift extra-ordinary requirements south, and subsequently would serve as a shuttle resupply between Subic and Haiphong with mail, freight, and passengers; an ARS was assigned for potential salvage, tow, and rescue service; a hospital ship was assigned; and of course, the big decision was made that the staff itself would go in the USS AJAX, for the services of a repair ship was required. In addition, it was felt that the first team should launch this operation.

There was a flurry at the replenishment ports in Japan while ships topped off and departed, and the staff was busy writing sailing orders and trying to anticipate other requirements not yet cited, but sure to crop up after arrival on the scene. Arrangements were made with CTF 72 for seaplane mail service, Sangley Point in the Philippines to Tourane Bay in French Indo-China, three times per week. The cargo ship, USS UVALDE, then unloading on a pipeline run, was pulled out of the pipeline and ordered to Yokosuka and Kobe to load the special requirements of the "Passage to Freedom" operation, principally rice and medical supplies and equipment. A branch Navy Motion Picture Exchange was put on board the UVALDE with 50 sea prints.

Service Squadron THREE was now prepared for the following: provisions, regular and special; fuel and lubes of the usual fleet oiler load; movies; repair services; medical services and supplies; towing service; and freight service. However, at this point it was decided to further exploit the capabilities of our ships. The USS AJAX (AR6) was assigned a variety of missions, among them general and technical stores issuing functions, and she took on a load of beer and other potables, for recreational facilities would have to be established. Extra deck loads of drummed motor gas were loaded on the oiler and the AR. Although it was most undesirable to carry large quantities of gasoline in this way, it was in constant demand for boats and had to be resupplied regularly.

In the meantime, messages had gone back to the Chief of Naval Operations recommending the reactivation of the PASIG, a water distilling ship, to meet the water requirements. Two idle tankers in Japan were ordered for an immediate cleaning (they had previously loaded with leaded products, thus sand blasting to the bare metal was involved) and they were then loaded with water and dispatched to Tourane in all haste.

A stop at Subic Bay in the Philippines on the way permitted the AR, already loaded to the guards, to take on additional lumber, medical supplies, movies, and tropical clothing. Mail and fleet freight, already beginning to accumulate there, was also picked up.

On the morning of 23 August COMSERVRON THREE arrived at Tourane Bay. Immediately the following message went out, "GANO'S GARAGE, GENERAL STORE, GAS STATION NOW GOING X YOUR PATRONAGE SOLICITED." Some TF 90 ships had already arrived, and the lift had begun, but none of the ships had made a complete round trip by that time. About 10,000 refugees had been picked up, and the "PASSAGE TO FREEDOM" operation was in high gear.

From communications received while on the way to Tourane it was evident that two things were on the minds of the people already there beyond the basic essentials of existence. They were mail and a recreation facility. Mail call was sounded as the hook of our flagship went down. Shortly thereafter, survey parties in boats searched the shore line for a beach site. Within 2 days a beach recreation area was in operation affording good swimming, warm sand, cold beer, and soft drinks.

The terms of the cease fire precluded any liberty north of the 17th parallel and the terms of our official exchange rates and inflation precluded for all practical purposes any liberty in Saigon. As for Tourane, that was a sleepy town completely taken over as a garrison and being devoid of "any place to go," was as dangerous as a rattlesnake with some evidences of international hard feelings flaring up, with almost nightly reports of unexplained sniping.

Over the next several days and weeks our ships came in. For the first week, however, the AR was virtually all things to all people. She served as a stores issue ship for all types of provisions, general, technical stores; provided mail, fleet freight, and receiving station services (among them the construction of large camels for the alongside operations); operated both an officers' club and a recreation beach; and served as a fleet movie exchange. The AJAX really proved how versatile one of our big tenders can be.

As our other ships came in, they assumed their normal duties in an inport replenishment group, and some of the extra details were farmed out from the AJAX. The UVALDE was kept out of her normal pipeline operation and she was the issue ship for the "exotica" we carried. It was not long before we had settled into a pretty normal routine of inport replenishment.

One procedure was quickly adopted, which for effectiveness procedurally and psychologically, was our most outstanding tool. That was our boarding call. Our boarding officer invariable went up over the side of every ship entering as its anchor went down. And with him went a mail clerk and the ship's mail. From that moment we had them on our side! All the small details of their routine for the next 24 hours were settled on the spot. All the questions that reams of correspondence couldn't have covered were disposed of, and the ship was then able to start its liberty, plan its working parties, request any special services, and be referred to the local expert for their special problems right then and there. That prompt and unfailing service was our best "gimmick," the secret of our success.

Water was our biggest problem, but a water barge was brought over from Subic. By utilizing any extras from ships with small excesses and by rationing these excesses, we eked out our existence. The local water was so heavily contaminated that even after chlorination it was feared and so never used. The water tanker and the TAW, the water distilling ship, eventually took over the solution of the problem. The TAW had to be reactivated on the West Coast and finally arrived on the scene in early October.

Prior to departure for Tourane, a request was made to Commander-in-Chief, U.S. Pacific Fleet, for information as to the type of currency to be used during the operation. It was not until after arrival in Tourane that firm currency instructions were obtained. The policy to use dollars, not MPC, and deal in Piastres only at the official exchange rate of approximately 35 to 1 was essentially troublesome, since the free rate varied from 58 to 70 to 1. Since prices in Indo-China were based on the true exchange rate, rather than on the official rate, it caused the military to pay double for everything purchased.

Indigenous stevedores were available. They were very low grade labor, predominantly women, and it appeared altogether barbarous to watch tiny little women, many to all appearance at least seven months pregnant, lifting hundred pound sacks of rice around the holds of our ships.

There is no substitute for a detachment of a cargo handling battalion in an operation such as this. Such a unit would be available for handling of ship to ship transfers, as well as to assist in the efficient handling of fleet freight.

A working platform, such as a Covered Lighter (YFNB) might have provided, was badly needed for our mail, movie exchange, fleet freight and receiving station functions. None of our ships have much free deck space, and the long trips up and down ladders made rough jobs rougher. However in extenuation, Service Squadron THREE wanted to take into the area a minimum of ships or equipment that couldn't get itself out come some emergency evacuation or before some change in politics or the weather.

While the USS AJAX was available, the coordination and utilization of repair facilities presented no problem. The COMSERVRON THREE Staff Maintenance Officer screened repair requests and insured maximum repair accomplishment with available facilities. After the departure of the AJAX, the SPHINX (ARL-24), was the only repair vessel available in the area. The SPHINX did not come under the operational control of CTG 92.3. Without a staff maintenance officer during this latter period, and lacking operational control of the SPHINX, coordination of maintenance work was not handled as efficiently.

There never seemed to be enough boats, although extras were taken and boats were borrowed from the amphibious vessels. This problem was made more acute by a very simple thing done in the preparation stage - to give plenty of elbow room the anchorages were placed at 1,000 yard intervals so every boat trip was accordingly lengthened. This was corrected when time was available to make a revised anchorage chart. Then too, there were so many special requirements for boats ordinarily accomplished by yardcraft - boats as tugs, boats as garbage lighters, and boats for a hundred other services not generally predictable.

Sangley Point's VR23 detachment, and CTF 72's patrol planes gave us excellent mail service initially. Incidentally, the patrol planes made us unlimber and relearn an old but not often used replenishment procedure, gassing seaplanes from oilers. The oiler had forgotten they were supposed to know how, and the aviators had too, but it worked, and beautifully.

During the latter portion of the operation mail proved a problem. The uncertain scheduling of air transportation created unnecessary delays in the make up and delivery of mail within the area and at the back-up point. The AKL's arrived from Subic with airmail on board when there was space available in mail planes. Delivery of registered and other security mail was hampered by lack of security at French airfields and, therefore, such mail had to be transported by ship.

A survey conducted among the ships in the area showed delays of from three to six weeks in the receipt of official and registered mail by the ships, and checks and money orders by the dependents. Some of the ships reported no receipt of any mail for periods in excess of thirty days. Others reported only three deliveries within the past two months. Mail was also reported received out of sequence. Mail was at times carelessly routed. For example, several bags were received on the 10 November flight marked "MSTS MIXED" that contained nothing but mail for ships of all types, destroyers and carriers included, none of which had ever been involved in the French Indo-China operation. All of these complaints resulted in mail tracer requests, thereby adding to communication traffic. Mail misrouted created additional work and further delays.

The absence of a medical officer on the staff, or on any ship of the task group after the departure of the USS AJAX, made it necessary to depend on any ship in the anchorage that might have a doctor on board. If there had been no doctor in port, emergency cases would have had to be moved from the area, or held for the return of a doctor. In addition to emergency cases, all

ships without medical officers requested medical assistance from the logistic support group. Without medical advice immediately available, considerable time and effort was expended in procuring medical services for these ships from casual sources.

A surprising thing was the volume of passenger business experienced. From twenty to fifty transients of one sort or another were moved on an almost daily basis. This emphasized the fluidity of personnel business in logistics.

As time went on, a pipeline operation was instituted from Subic using light cargo AKL type vessels, and the flow of special requirements beyond the capabilities of the replenishment group and airlift was assured. Dimension lumber for the construction of camels, tropical clothing, motor gas, and drummed lubes were in almost constant demand, as well as some technical parts from the AKS(T), the technical issue ship, at Subic. Nevertheless, the AKL types, shuttling between Subic and French Indo-China could not carry many passengers and were slow. Failure of rear area coordination was particularly frustrating, as with the mail.

The swells in Tourane Bay were constant and camels were ground up as fast as they could be made. Even the sections of pontoon barges were holed and put in sinking condition by the constant butting of the ships against each other. On more than one occasion the oiler had to get underway and refuel ships at sea rather than alongside in port.

Hospital ships, HAVEN and the CONSOLATION, were made available. The HAVEN proceeded to Saigon, picked up several hundred French soldiers and evacuated them to North Africa and France. The vast majority of these soldiers, besides suffering from actual wounds, were suffering from the usual tropical diseases and parasites. The CONSOLATION joined the inport replenishment group and it was amazing how many ills immediately cropped up that could only be attended to on board the CONSOLATION. It was fortunate, however, that our personnel never contracted any of the epidemic diseases, and the CONSOLATION was released after a short stay to go to Inchon, her normal station.

The pipeline reefer was deployed to Indo-China periodically to top off the small AF on station. The CASTOR, a general stores issue ship, was brought over from Subic Bay to give the ships a larger opportunity to draw general stores. Although the CASTOR was useful, the volume she issued was relatively small, a tribute to the work of our inport replenishment groups, and to NSD, Yokosuka and Fleet Activities, Sasebo, who had done marvelous jobs of topping off the ships on short notice before departure.

As a matter of fact, with the exception of the high usage items and the epidemic control supplies, none of the issues were up to expectations. The French provided rice for voyage feeding, and every family brought straw mats. However, a large amount of our exotic merchandise was used up in gift packages which were presented to the passengers upon leaving the evacuation ships as a gesture of goodwill. A bucket, a small package of rice, a mat, a set of chopsticks, cigarettes, a candy bar, and a cake of soap were presented with appropriate ceremony, labeled with "Passage to Freedom" stickers.

Communications caused some difficulty, particularly in the early stages of the operation. A considerable volume of traffic relative to logistic requirements was received until the ships learned that their needs would be taken care of amply and promptly subsequent to the boarding call. Our harbor logistics circuit was an invaluable aid as it always is, and it was used to the fullest extent.

The Military Sea Transportation Service Cargo Ships (TAK) with partially Japanese crews, presented a special logistics problem. They arrived stocked with a theoretical 90 day supply of Japanese provisions, but the storage life of the more perishable items was exhausted

before the 90 day period had elapsed. For ships with foreign nationals as crew members, special replenishment plans must be set up in advance. It was found that the fact that these crewmen do not like or work well on American rations necessitated special planning.

In spite of the many operating difficulties encountered, such as worn out reefer ships, personnel shortages, boating shortages, a force spread between four operating ports, communications difficulties and the like - the job was accomplished. Up until 15 November, when the logistic support force, and the remainder of the USN ships were withdrawn, 173,311 passengers had been transported and 58,249 tons of cargo had been lifted, along with 5,791 vehicles of all types.

A significant message from CTF 90 to CTG 92.3 is quoted:

> "YOU ARE RELEASED 15 NOVEMBER X THE EFFECTIVE SUPPORT OF TASK FORCE 90 BY TASK GROUP 92.3 MADE POSSIBLE THE PASSAGE OF APPROXIMATELY 170,000 SORELY OPPRESSED REFUGEES FROM THE VIET NIMH TO FREEDOM IN THE SOUTH OF VIET NAM X WE HAVE LACKED FOR NOTHING IMPORTANT IN THIS REMOTE AREA THANKS TO YOUR OFFICERS, MEN AND SHIPS X WELL DONE."

It is concluded that the experience gained in the "Passage to Freedom" operation has highlighted many required additional refinements in logistics techniques which might have otherwise remained obscure.

* * *

CONSISTENCY PROBLEMS IN THE MILITARY SUPPLY SYSTEM[1]

Oskar Morgenstern
Princeton University

This paper shows a simple, feasible model of a military supply system composed essentially of several Overseas Bases, each supporting different activities; the bases are supplied by a domestic Supply Center. The problem is to establish procedures that guarantee a functioning of the system. This is achieved by determining those factors that should enter into the planning of the component parts, and to eliminate others that are sometimes used to the detriment of consistency and efficiency of the system. The section headings are as follows:

1. Introduction
2. Supply Center - Bases - Activities
3. States of Deficiency
4. Priority Orderings
5. Procurement Time
6. Continuous Transportation
7. Discontinuous Transportation
8. Deadline Delivery Dates
9. Resume
10. Minimum Processing Time at the Supply Center
11. Follow-Ups
12. Peacetime and Wartime Levels
13. Policy Comments

1. INTRODUCTION

In the following are set forth the essential elements which are required to bring about a shipment from a domestic Supply Center to an Overseas Base. This paper does not give a description of current practice, but sets up a normative scheme which, if adhered to, would assure a rational, feasible system. The principal task is, first, to find arrangements of activities that are free from inner contradictions and, second, to select the one that is judged most appropriate under the various constraints of expense, speed, vulnerability, etc. For each system considered there arises separately the problem of optimal operation. We are only concerned with the first task by describing the structure of one of many feasible systems. A detailed comparison of the one presented with those used by the Navy, Air Force, etc., would be instructive, but this will be done merely in some specific instances, leaving a fuller analysis to another opportunity.

Since the various military services use their own terminologies it is not possible to conform to all of them. We have chosen to use our own terms, each of which will be explained sufficiently so that misunderstandings should not arise.

---

[1]Most of this work was done at Princeton University under Office of Naval Research Contract N6onr-27009. An earlier version, under the same title was circulated (14 July 1954) as RM-1296 by the Mathematics Division of the RAND Corporation, where it had been finished.

For valuable comments received on RM-1296, the author is grateful notably to Rear Admiral W. G. Schindler; Commander E. K. Scofield, SC; Commander G. L. Voegeli; Lieutenant Commander J. C. Busby, SC; and Dr. J. Laderman. However, the author assumes sole responsibility for the views expressed in this study.

The system we are going to describe is exceedingly simple; it could easily be made more realistic by increasing the complexity of the situations analyzed. This shall not be done here. It is, however, important to realize that every supply system, whatever its complexity, is embedded into a larger one with which it interacts. For example, the Pacific supply system of the Navy is embedded into the Navy's entire system, and the latter is embedded into the whole economy of the United States. Imperfections of the behavior of a particular supply system which are sometimes attributed to its design are often due to this embedding relationship and the interactions it produces. This is particularly important in military supply systems which are frequently subject to sudden changes in strategy decisions and, in war, also to combat effects, both of which may impose a strain on the larger system.

Clear and exhaustive description of even the simplest set-ups is the first and indispensable step towards over-all optimization. Description of the kind attempted here has to precede the construction of rigorous mathematical models. These models are, however, required in order to test the working of the system computationally, discover optimal procedures, and to proceed to the construction of more satisfactory set-ups.

A supply system shall be termed "consistent" if the various component parts, in behaving according to general rules which are part of the set-up, do not interfere with each other. This requires a consideration not only of physical installations, such as bases and depots, but also of the distribution and direction of information flows, etc. While a common sense approach to these problems will lead fairly far, a thorough exploration can only succeed when the rigorous model has been developed.

2. SUPPLY CENTER - BASES - ACTIVITIES

There exists one domestic Supply Center and at least one Overseas Base. The latter is supporting one or several activities, situated in a Theater; each activity is running at a certain rate. There exist expectations about these rates depending upon the composition and the general plan of operations of the Theater. The Overseas Base may be supply ships and the activities may be components of the fleet; or we may deal with land bases and airplanes.

The support of the activities at current and expected future rates requires stocks on hand at the Overseas Base and a flow of shipments to it from the Center and from the Base to the activities. (We omit the possibility of the Bases dealing with each other and of the activities dealing with each other or directly with the Center: if they do it always, they function as Bases; if they do it on occasion, while normally dealing with their supporting Overseas Base, they complicate the picture beyond the degree we wish to admit in this scheme. We likewise omit the even graver complication that there are several domestic Supply Centers which can draw on each other for supplies and otherwise interact.)

An equilibrium exists in the system when the Overseas Base is able to meet in time the demands from the activities, and the Supply Center the demands from the Base. This is achieved when the outflows from the Base's stocks are matched over time by inflows from the Supply Center. The stocks of the Base may therefore sometimes fall below, and sometimes rise above, the level the Base in the average considers normal, given the expected rates of outflows and inflows.

In a steady state the items $x_1, \ldots, x_n$ in the system are given and will remain fixed, although the quantities in which they are held in stock and the rates of their consumption may vary within bounds. However, it is typical for supply systems that due to technological changes always new items $x_{n+1}, x_{n+2}, \ldots$ are introduced and others already in it are made obsolete.

For the new items rates of consumptions on which to base expectations do not exist but have to be assumed in order to determine stock levels. This is particularly difficult in military supply systems when the new items are weapons for which no combat experience has been gained.

Over time the balance must hold item for item except where substitution exists. Substitutability, i.e., the possibility of using one item instead of another, sometimes at least temporarily and/or for restricted quantities, must first be recognized and second be exercised by the Overseas Base (and the Supply Center), lest an over-all level of balance is maintained that may be too high to be warranted in terms of economy of operation. However, we are only concerned with feasibility at present and not with optimality of the operations. Therefore we continue in this vein only.

Whether substitution is possible at the Overseas Base depends on technological interchangeability of the items (which may be only temporary), their relative stock positions, their expected rates of depletion and outstanding orders with their expected fulfillment. Introduction of substitution into the system requires a higher degree of information than is necessary when stock positions are considered separately. Sometimes this information may be of a very commonplace knowledge (e.g., peas can be used instead of beans, etc.), or it may require detailed technological training (e.g., when one electronic tube can do for another). The supply system may operate also by allowing for substitution of one item for another by the Supply Center; this could be restricted to technological interchangeability of items (e.g., it will send another tube than the one ordered; but this would be considered fulfillment of the order). Substitution will be resorted to if the original demand is of greater urgency or importance than the demand - if any - for the actually supplied item. That is, substitution will take place when the risk of reducing the stock level of the item from which the substitution is made is considered less important than the consequences of non-fulfillment of the order. The role of substitution will increase under critical shortages, especially in war time, and when economies of operation are important.

3. STATES OF DEFICIENCY

In an Overseas Base, in the normal course of events, for any particular item, already in the system, a true State of Deficiency may be expected to arise. This state is defined as the inability of the Overseas Base to provide from its stock or expected delivery a supported activity with a specific item in a specific quantity at a required date. It is the duty of the Overseas Base to take measures to avoid the occurrence of a State of Deficiency or to minimize the implied losses when they cannot be avoided. This is achieved by being supplied in all items properly, where "properly" means the stock level corresponding to the expected rates of demand from the activities. The State of Deficiency, in order to convey meaning, has to have a date attached showing when it begins; it lasts until it is remedied by receipt of the necessary quantity of the item in which the deficiency occurs, or by cessation of the need, for example, due to change in the operating levels of the activities.

To form an expectation about a true State of Deficiency it is necessary for the Overseas Base to know: (α) the stock position, (β) outstanding orders (with average lead time, i.e., normal time interval between placement and expected fulfillment of the outstanding order), (γ) current and expected rate of consumption of the item by the supported activities, and (δ) substitutability among different items. Of these (α), (γ), (δ) is information involving only the Overseas Base, while (β) is information that can be based on past experience of the Base as well as on current information from the Supply Center. This expresses that past behavior of

the system is essential information for determining future behavior; this fact has to be incorporated carefully into the structure to be set up.

A true State of Deficiency is not an unalterable condition in the system in the sense that it will necessarily occur at the conjunction of even the same values for all the factors enumerated under (α) to (γ) above. In other words, the Overseas Base will depend on the level at which it is required to supply the activities. These may be able to get along with much less material than authorized, they can be cut back in their consumption, or they may be allowed to increase it. Thus, the true State of Deficiency is, at least for many items, relative to the authorized level. For others, e.g., parts of ships' machinery, there may be no admissible variation.[2] These changes are, of course, not identical with those due to variations in the military worth of the Overseas Bases, although there arise connections between these two.

It is clear that the occurrence of a State of Deficiency is one thing; its recognition and the placement of an order in time to avoid its occurrence is another. While it is the duty of the Overseas Base to avoid the occurrence by timely order, a State of Deficiency may arise for reasons not foreseen in the scheme (e.g., enemy action, fire, etc.), or because of faulty information, mistakes by the Supply Center, the transportation system, etc. There is nothing logically wrong in having a State of Deficiency even before an order has been placed, while it is logically absurd to have a Deadline Delivery Date before the order has been received by the Supply Center[3] or for a time less than the minimum it takes to fulfill an order.

There are different kinds of States of Deficiency that should be distinguished. We do not attempt, at this point, a complete enumeration of possibilities but mention only these: (1) ordinary: when the item is carried by the Overseas Base; (2) special: when the item is not carried but, because of other circumstances, the Overseas Base is to supply the item. In case (2) the State of Deficiency will as a rule (but not necessarily) be anterior to the receipt of the order. The determination of the importance of one or the other type of State of Deficiency depends on the concrete circumstances at the Base and cannot be made a priori.

The notion of a State of Deficiency carries over, mutatis mutandis, from the Activities to the Overseas Base and finally to the Supply Center. The latter, too, has the duty to avoid the occurrence of such states principally achieved by stock control. The probable lead times that are relevant are those of the manufacturers. This aspect shall here not be pursued further.

States of Deficiency, whether actual or expected, are remedied by two means. First,

(a) Locally (i.e., at the Overseas Base):
  (1) by repair
  (2) by substitution
  (3) by cannibalization
  (4) by provisioning: (α) purchase
    (β) requisition without payment (e.g., in wartime).

The deficiencies thus remedied are called pseudodeficiencies because they do not immediately involve the Supply Center directly, even though they may require time and means in order to be remedied.

---

[2]This is intimately tied up with the question of whether and how organizations can be "compressed" without suffering vital damage. It has been discussed in: O. Morgenstern, "The Compressibility of Organizations and Economic Systems," RAND, RM-1325, August 1954, and will be taken up in another publication.

[3]This is frequently the case at present. More about DDD's below in Section 8.

Comment on the four cases can be brief, except regarding (2). As to (1), it is obvious where it applies; (3) and (4β) are typically emergency operations mostly of relevance in wartime. (4α) is a procedure recommending itself especially for items of small value and frequent occurrence.[4] Furthermore, (1) and (3) will only be of interest if a State of Deficiency has already occurred since, for example, nothing needs to be repaired before it has broken down.

Remedying a pseudodeficiency may cause a true, i.e., locally not further remediable, deficiency elsewhere on the Overseas Base [except under (4)]. Local action will be justified if the deficiency avoided is greater (i.e., worse) than the one caused. There may even be no ultimate genuine deficiency, for example when only the transfer of a unit of a substitutable item is involved, that is in stock, well above the reorder point. The consequence of this action is merely to advance in time the expected date of the state of deficiency of the latter item (by increasing the expected rate of consumption). That is to say, the substitution produces effects equivalent to a larger than expected demand for the item used as a substitute.

If substitution is permitted great economies of operation are possible as well as a speed-up of the supply of urgently wanted material. But the consideration of substitution requires a higher level of information; some of it will be trivial, other quite technical. Substitution will not always be mutual, i.e., item x may be substitutable for item y, but not vice versa, etc. These relationships are familiar in economics and should be carefully exploited to make a supply system more efficient.

Normally, after several substitutions or for the other reasons enumerated under (a), we arrive at a point where a true deficiency is expected to occur. Then we have the second case, where the only remedy is:

(b) Nonlocal, i.e., at the Supply Center by means of:

(1) requisition with

(α) specification in kind:

(α') rigid, (e.g., caliber of ammunition)

(α'') limited, (i.e., allowing for some substitution; e.g., one food for another).

(β) specification in quantity:

(β') fixed, (technologically conditioned, e.g., one periscope; a second one is useless or even a nuisance, none means totally missing the order)

(β'') maximal, (if less is shipped, this is a partial fulfillment of the order and not necessarily meaningless, e.g., less food than demanded, though more, especially the demanded quantity, would be preferable. More than demanded may be embarrassing if the additional quantity cannot be stored (e.g., perishable), or only at cost, or ties up shipping facilities (by requiring unloading), etc.)

A combination of some α with some β is always necessary, and several forms of combinations are possible. E.g., (α') and (β'') may go into one order; so, for example, when 1000 tons of a particular, rigidly specified ammunition is demanded. If 900 tons are received, a partial fulfillment of the order has occurred; but if 1000 tons of a wrong caliber was shipped a mistake has been made, making the shipment worthless as far as this order is concerned.

---

[4]Great savings in any supply system are possible in this connection, especially when the individual item costs little but the processing of the order (indirectly) very much. In this case it is simplest to ship money and not to insist that, for example, each pencil be exactly of the same make overseas as at home!

If resort is had to (b) an order (or requisition) arises that is transmitted to the Supply Center. This involves a transmission time of the order, which, however, shall be neglected in the following, assuming it to be negligible in view of telegraph or even air mail. This time interval is, however, often not a negligible quantity.

It is feasible to include in the design the possibility that Overseas Base A communicates with Overseas Base B in order to obtain a needed commodity or its substitute before placing an order with the continental Supply Center. Similarly the latter, instead of fulfilling an order directly, could be authorized to direct shipments between Overseas Bases. In the latter case A need not know that B (which may be nearer to it than the Supply Center) could supply the needed commodity, while the Center has to know or be able to find out. Since we are, in this paper, primarily concerned with feasibility, and not with optimality, of operations this reference will suffice. In actual experience some undesirable practices have been observed in that material has traveled unnecessarily between Overseas Bases over great distances, the decision to ship made by minor officials possessing wholly inadequate information about the state of the supported activities, projected plans, etc. On the other hand, in the development of mobile support of the fleet in the Pacific the simple device of direct radio communication among supply ships in a limited area has proved of great value, avoiding unnecessary requisitions from the Supply Center.

4. PRIORITY ORDERINGS

The home Supply Center receives the order together with many others from the same or other Overseas Bases. If they arise from the same Overseas Base, they should at the time of their arrival contain two pieces of information: (a) expected Date of the State of Deficiency in case the current order should remain unfulfilled as of that date and (b) priority (or "ordering of importance of the expected deficiency to the particular Overseas Base").

If the Supply Center is only dealing with one single Overseas Base the priorities assigned by the latter will be those that should govern directly the operations of the Center - as far as priorities are concerned. (We shall see presently (cf. 7) that they can never exercise exclusive control.) We assume that each Overseas Base is able to assign priorities to its orders. This means that the Base would be able to say that a state of deficiency of x is worse than one of y (although x may occur at a later date than y!) and that consequently the order for x is given a higher priority than that for y. This is a far-going assumption, but it shall be made.

It is possible to classify by commodities or events when priorities of different rank are to be assigned. This is done in the Navy (cf. ComServPac Instruction 4220.2B, February 1954). It is however, almost impossible to adhere to these classifications and to avoid serious abuses.[5] Thus, the responsibility will essentially rest with the Base commander that only meaningful priorities are assigned to the orders going out to the Supply Center. An important characteristic of any working system is that each component has confidence that every other component behave according to the rules. A further corollary is that high priority orders be speedily

[5] Cf. E. F. Hollingshead: "The Application of Statistical Techniques to Management of Overseas Supply Operations" in Naval Research Logistics Quarterly, Vol. I, p. 82, especially the examples given on p. 87. There exist worse cases than those mentioned there.

transmitted and contain enough detailed description so as to reduce difficulties of identification of the ordered items below the risks of misinformation compatible with low priorities.[6]

If the Supply Center is simultaneously dealing with several Overseas Bases (the typical case) the priorities assigned by different Bases are important information for the Center but do not any longer justify their direct guiding the operations of the Center. In other words, a priority A from Base 1 is equal to a priority A from Base 2 if and only if the military importance of Base 1 is equal to that of Base 2. This information must be made available to the Supply Center by the theater commander if the different bases supply activities lying within the same theater. Should the Center supply more than one theater their importance has to be ordered by still higher military authorities. These are frequently changing military decisions that enter as data into the supply decisions.

Each Overseas Base is confronted with the same kind of problem if it has to support more than one activity. So while a Base can set reasonable priorities for the demands of activities it supports, only the Supply Center can determine the relative values of the priorities originating from the different bases. No Overseas Base can expect that the priorities it assigns to its orders be maintained by the Center and not be reclassified. Even after the Supply Center has established its own priority rating for the orders received, further information is required (e.g., stock positions, manufacturers' lead time, etc.) before it can be decided which orders from which Base have precedence over which others.[7] It is clearly impossible for any one Overseas Base to perform this ordering of operations. Even when only one single Overseas Base is involved the entire information required by the Center would have to be passed on to the Base - obviously wasteful, if not technically impossible.

We assume here that the Center (and for its more restricted tasks: the Bases) can actually order different priorities from different bases. This is not a simple task; it involves difficulties familiar from utility theory which are particularly significant in the absence of a monetary measure. We can, however, assume the feasibility of such orderings; in the absence of precise and clear instructions about the relative worth of different military operations that can be given the Center, common sense and military experience together have to come into play. At any rate, it is not possible to design a supply system of even a modest degree of complexity that would not be confronted with this need of ranking priorities.

A great additional difficulty arises from the continuity of operations, both of the Overseas Base and the Supply Center: in the former, true States of Deficiency are expected to arise or have already arisen accidentally in many fields at irregular intervals; and in the latter, the Supply Center receives many orders in a nonplanned, but not absolutely random manner in a practically continuous stream. The operations of the System have no beginning and no end; there is no obvious time period over which to maximize results (or, for example, minimize

---

[6]At present in the Navy operational and logistic messages are intermixed on the same circuits; thus, they interfere with each other. Often greater detail in the description of high priority orders would have eliminated serious misunderstandings, but the rules were against more detailed description. It should be studied whether the setting up of separate logistic circuits (with the proviso of Section 12 below in mind) would not substantially improve the information flow and therefore the working of the system as a whole. In this paper we abstract, however, from these matters.

[7]It may be observed here that there exists no real theory of priorities; especially the working of priorities in systems where also prices prevail and localized decisions are frequently possible, is only imperfectly understood. Here is an important field for research.

costs). This also allows for no permanent, stable ordering of the importance of the different States of Deficiency. For example, a later one may be more important to the Overseas Base than an earlier which may well be in execution without possibility of stopping it, etc. Yet the Supply Center has to adhere to some principle in executing the orders and has to set up a daily plan of operations. Being unable to do all things at once it has to do them sequentially though in many parallel streams. In business this problem is comparatively simple: one may merely order chronologically, or by monetary amounts involved, or by profitability of the individual transaction. But here the troublesome question of military worth is raised which the Supply Center (or, indeed, the entire supply system) cannot avoid answering even if only in the crudest manner by its own actions. Thus we have to reserve for the Supply Center the right to rearrange any time sequence the Overseas Base may have indicated for its own orders. The Center will be guided by these factors: availability of the material, shipping dates and space, simultaneous orders from other bases and over-all importance of different fleet or theater operations. [The possibility exists to provide for changes by the Overseas Base in priorities it had assigned to earlier orders together with the placing of a new order so as perhaps to expedite the latter. This case shall not be considered explicitly because of its complexities in the implied information pattern.]

5. PROCUREMENT TIME

It is the task of the Supply Center to place the ordered item in the desired quantity at the Overseas Base before occurrence of the expected date of the State of Deficiency. This will not always be possible; clearly not if the State of Deficiency is earlier than the receipt of the order. More generally, therefore, the Supply Center has to provide the requested item in the shortest possible procurement time, the latter to be defined in the following.

The dates of receipt of the order and of the expected State of Deficiency define a time interval of n days. The Center has to compare this number with the following:

(a) The pure transportation time from the Supply Center to the Overseas Base (for the type of transportation, vessel, air, etc.) excluding waiting, misdirection, etc.

(b) The time from the receipt of the order to the next sailing date to the Overseas Base on the type of transportation admitted for the kind of priority attached to the order (viewed in the whole priority picture). Only a sailing with accommodation after the lapse of time (c) counts; earlier are skipped unless the Center-assigned priority makes one available.

(c) The minimum processing time at the Supply Center (for the particular item, its Center-assigned priority, quantity, stock position, possibly probable lead time and the level of over-all activity at the Center; i.e., its congestion).

To determine the minimal procurement time we have to consider that there is either continuous or discontinuous transportation; in the former case (b) is irrelevant because there is always a sailing date of the appropriate kind of transportation. We discuss this eventuality first.

6. CONTINUOUS TRANSPORTATION

With continuous transportation the ordered item will be expected to arrive at the Overseas Base in as many days as correspond to the sum of minimum processing time plus pure transportation time, stated in days. The planning of the Supply Center is then entirely dependent on its own facilities, considering also the probable lead-time of an order from the manufacturer should the Center be out of stock.

Any specification of a desired arrival date (or deadline delivery date (DDD)) of the shipment earlier than the number of days equal to the sum of minimum processing time plus pure transportation time is immaterial and has no operational value whatsoever. Such a date should never guide any operations; it suffices to demand from the Center that it achieve processing in minimum time. Indication of a later date allows the Center to operate with less effort than the minimum processing time represents (it being minimal); the effort may be costly in terms of money as well as by overriding other orders causing them to be delayed. Indication of a desired arrival date would require for the Overseas Base to know the minimal processing time and the pure transportation time. While the latter can be known with fair accuracy, the Base can only have a rather vague knowledge of the former, since it does not know the Center's stock position, the possible lead time of the manufacturer (if out of stock at the Center), the congestion at the Center, the priorities of other orders originating from other Overseas Bases, etc. It would be possible to compute expected values for most commodities and to inform the Overseas Base currently of changes, but the burden on all would be formidable and entirely unreasonable. Without this minimum requirement of accuracy the use of a DDD is only confusing.

For the case of continuous transportation the sum of minimal processing time plus pure transportation time is the Minimal Procurement Time in which it is possible for the Center to get the ordered commodity actually into the possession of the Overseas Base. (We have abstracted from loss of time in sending the order and end the analysis at this point, also neglecting final unpacking and other processing of the shipment at the Base.)

Continuous transportation can be assumed to exist for many cases. It obtains for the continental United States when parcel post can be used, commercial air freight with daily flights, etc., or when it is possible to charter transportation of the required kind on short notice, i.e., in such manner that the carrier is ready whenever the minimal processing time ends. The possibility of using parcel post is sometimes much neglected in supply systems, especially in view of the fact that most orders are for small items in small quantities.[8] Parcel post to the Far East is not continuous in the above sense because the sailing dates are spaced considerably apart, but even so it is available.

There is, of course, the requirement of sufficient capacity of the transportation means that are assumed to be continuously available. For normal operations the post office has the capacity to handle all parcels that may be shipped by a Supply Center. But there are situations where only the appearance of continuous transportation may be given. In those cases accumulation of shipping space may be required before the necessary capacity is obtained. This would then be one form of discontinuous transportation, i.e., the situation where transportation is not available at any arbitrary moment of time.

7. DISCONTINUOUS TRANSPORTATION

If there is discontinuous transportation - obviously the normal case - considerable complications arise: Both the Overseas Base as well as the Supply Center require a higher degree of information than they possess now in most supply systems. The discretely spaced sailing and flying dates—whether they follow a regular rhythm or not—become the focal points

[8] We are, in this paper, not discussing the admissibility of these orders, their frequency, smallness, etc., or the relation of value of the item ordered to the cost of processing the order, etc. Great economies are possible in this field by comparatively simple changes in operational procedures governing the system. Here we are only concerned with the system, not its handling.

for determining what volume of material can be sent when to what points. These are three variables entering newly into the picture; they are not present in the case of continuous transportation. We shall deal principally with the "when" only. A ship sailing date is the indication of a future day at which a particular "vessel" of a certain type (tanker, general cargo, airplane, etc.) is to leave the Supply Center with a definite destination; it implicitly specifies scheduled and probable arrival dates overseas.

The sailing date of a vessel not suited for the kind of commodity, or with no accommodation for the shipment considering the priority assigned to it by the Center, has to be omitted as irrelevant. So the Supply Center has to spot the nearest available sailing date of the proper kind of carrier that is more days, or at least as many days advanced from the date of receipt of the order, as is the length of the minimum processing time. Earlier sailing dates are irrelevant because the shipment could not be put on these ships; later are interesting as planning factors (α) in case of a mistake or (β) if out of stock or (γ) because the minimum processing time may safely be exceeded as when a ship sailing later would still arrive at the Overseas Base before the expected State of Deficiency (which may, of course, not be the actual one.)

The Overseas Base to know - and consequently plan - how much in advance of the expected State of Deficiency to order from the Supply Center, can no longer rely on the (average) minimal procurement time for continuous transportation; it would have to know the sailing dates of vessels leaving the Center, plus have information about availability of space on them. The latter it will never possess; only in the exceptional cases when whole ships and shiploads (e.g., tankers) are planned for the use of a particular Overseas Base exclusively. Thus an Overseas Base can, as a rule, not know the Minimal Procurement Time in the normal case of discontinuous transportation. But it can form reasonable expectations within certain bounds based on previous experience and if it has information about the scheduling of ships. On the basis of these it can make a thorough probabilistic evaluation of this information.

From the above it follows that in the case of discontinuous transportation the Minimal Procurement Time is equal to the Minimal Procurement Time for continuous transportation plus the waiting time for the next available vessel (with accommodation). The latter amount may, of course, be zero, but in all likelihood will be larger than zero.

We note that pure transportation time is not necessarily a constant even for the same type of carrier. Variations are due to the inclusion of Overseas Bases being mobile, e.g., parts of the fleet and by moving with it affect the time in which supply ships can reach them.

Continuous transportation is obviously more efficient and convenient than discontinuous. It is also less likely to be available except for most operations of the Supply Center involving bases in the continental United States. To approach this higher efficiency it is necessary to coordinate the estimated outflow of cargo from the Supply Center with the future shipping space by giving the Supply Center some influence upon, if not complete control of, future shipping schedules. The discussion above was under the assumption that shipping schedules, volume of cargo space and destinations are set by a shipping authority upon which the Supply Center cannot act. A better design would be to let the Center make its estimated future demands for shipping space and routes known so that the shipping authority can adjust sailing schedules to expected loads. While this would make for smoother operations, cut down tidewater time and thus speed up the flow, it still would not dispense with the need for the Supply Center to deal with sailing dates as described. The difference would be, however, that these dates would be related to the work load of the Center. The influence of this change of the setup upon estimation

of pure transportation times is obvious. It is clear that if there are several Supply Centers in our scheme, or if there are shipping requirements that do not arise from within the Supply System, a single Supply Center cannot control ship movements. Thus, there will have to be a shipping authority allocating space and routes according to the relative importance of the various demands made.

8. DEADLINE DELIVERY DATES

For the Overseas Base to fix a Deadline Delivery Date (as at present customary in the Navy) expecting it to be fulfilled and to become the guiding principle of the Supply Center (as is actually the case) is meaningless without a type and quantity of knowledge in the possession of the Overseas Base they cannot have. To provide it would involve not less than transmitting the whole information of the Supply Center to the Overseas Base (which may be a supply ship following the fleet!). It is meaningful, however, and indeed necessary (as pointed out above), for the Overseas Base to indicate the date when a State of Deficiency is expected to occur, how important it is for the Base and then to leave the rest to the Supply Center.

As it happens in some actual systems, sometimes in one third of all cases, the Deadline Delivery Date has already expired at the date of receipt of the order at the Supply Center. Sometimes it has expired for a longer period than the total average shipping and processing time! A system incorporating such occurrences - not to mention the frequency! - is in that respect clearly illogical in its conception. These Deadline Delivery Dates, around which a supply system theoretically could be built, are deprived of all meaning when such occurrences are possible. The Supply Center cannot act into the past. Nor can there be, in view of this situation, much confidence that other Deadline Delivery Dates, however far in the future, are proper characterizations of the future states and needs of an Overseas Base. In another - very restricted - much smaller type of supply system a Deadline Delivery Date could be used, provided the Overseas Base was equipped with information to determine its feasibility. It is feasible if the time interval between the date of the receipt of the order at the Center and the DDD is greater than, or at least equal to, the Minimal Procurement Time. Using always a (feasible) Deadline Delivery Date is equivalent to controlling the operations of the Supply Center from overseas. Even such a system would break down if more than one base is involved, since each base would have to know all about the others and each would have to be permitted to make decisions superseding those of all others. This is self-contradictory. In any other system, such as some actual ones, the formulation of a nonfeasible demand or of one where feasibility remains unknown to the Overseas Base - as at present - is merely wasteful and should be abolished.

Finally, we mention that in actual systems Deadline Delivery Dates are stated together with a priority rating for one and the same order. This confuses the issue still more since no satisfactory and practical principles can be worked out—and consequently have not been worked out—combining these two indicators. New ad hoc rules are introduced to determine whether an early Dateline with low priority is more or less important than a later with higher priority, etc. This shows that this combination is not unique but arbitrary. When orders from several Bases are received by the Center the difficulties are compounded. All this becomes irrelevant, however, because the Deadline Delivery Dates are unworkable at any rate, while priorities combined with States of Deficiency can be made systematic and consistent.

On the other hand, it is reasonable without restrictions for the Overseas Base to inform the Supply Center of a State of Deficiency, whatever its date. I.e., a justified State of Deficiency

may arise at any moment, for example because of physical accident, fire, enemy action, etc. It may already be in the past, even antedate the writing of the order. Its importance can be evaluated. A State of Deficiency, which, as we know, has only meaning as from a moment of time onward and lasts until remedied, is an objective fact, even when expected; it is amenable to probabilistic expression and analysis. It can be checked and controlled at the Overseas Base and by inspection from outside. Verification of actions by inspection is very important in administrative setups. Deadline Delivery Dates are not in that sense objective; they contain strong elements of subjective judgment, opinion, even temperament, etc. These are not easily testable statistically regarding soundness. Their influence upon the supply system remains, in ultima analysi, obscure.

It is now clear that the substitution of States of Deficiency for Deadline Delivery Dates is not merely a verbal matter, a change in terminology. Each has a different operational meaning, each causes different acts and sequences of acts to be set. The first leads to consistent behavior, the second cannot do so and introduces administrative uncertainty and ambiguity into the system.

9. RESUME

We can now state these about the relation between the date of receipt of the order and the State of Deficiency:

If the State of Deficiency is prior to the receipt of the order there has been:

(α) an error in transmission of the order

(β) extreme variation in demand from activities supported by the Overseas Base, where "extreme" means a random variation beyond previously accepted bounds.

(γ) an accident at the Overseas Base, perhaps due to enemy action

(δ) mistake or mismanagement at the Overseas Base

(η) mistake or mismanagement at the activities supported by the Overseas Base

(ψ) demand for an operational item not carried by the Overseas Base

These are clearly distinguishable from each other and can be evaluated by usual standards.

The following diagram will summarize some of the preceding discussion. It is valid for the case when the occurrence of a State of Deficiency can be prevented.

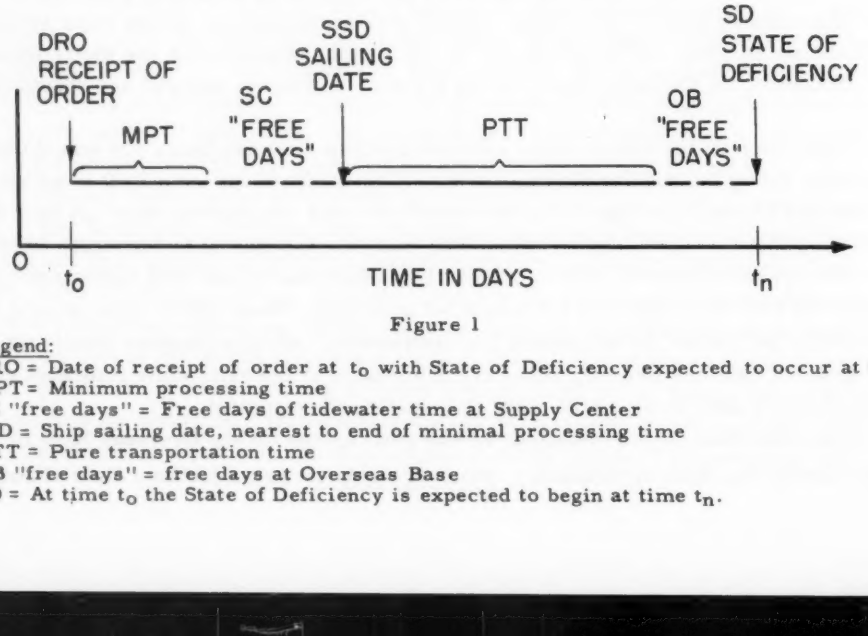


Figure 1

Legend:
DRO = Date of receipt of order at t_o with State of Deficiency expected to occur at t_n.
MPT = Minimum processing time
SC "free days" = Free days of tidewater time at Supply Center
SSD = Ship sailing date, nearest to end of minimal processing time
PTT = Pure transportation time
OB "free days" = free days at Overseas Base
SD = At time t_o the State of Deficiency is expected to begin at time t_n.

When the time interval from the date of receipt of the order to the sailing date is greater than, or at least equal to, the minimum processing time, the Supply Center can operate successfully as far as the sailing date is concerned.

If the time from the nearest sailing date to the beginning of the State of Deficiency (i.e., pure transportation time from sailing date on) added to the previous number of days is smaller than, or equal to, the time between the date of receipt of the order and the State of Deficiency, the Overseas Base will be completely satisfied[9] and a State of Deficiency will not happen. (We set the date of receipt of the order by the Supply Center and the date at which the future State of Deficiency is first determined to happen by the Overseas Base or becomes known to it as having already happened, as equal to each other. This amounts to omitting any time interval for transmission of orders.)

When a State of Deficiency occurred earlier, i.e., before the date of receipt of the order, or is expected to occur in a number of days which is smaller than the sum of minimum processing time plus pure transportation time plus free days of tidewater time at the Supply Center, there is nothing for the Overseas Base to do but to suffer the implied loss and to wait. This waiting (in silence) is justified by the general assumption that the Supply Center is performing its duty to ship as rapidly as possible and by the fact that the Overseas Base has no further control over the situation after sending the order, stating the priority indicator and the date of the State of Deficiency. Depending on the over-all situation at the Center (its "congestion") and the shipping space, the Center may be able to substitute faster transportation (e.g., air) for slower, etc. Again, the Overseas Base is unable to judge these possibilities.

Since a higher degree of information of the component parts of the supply system is desirable if it helps the parts to make better decisions, the Supply Center should provide the Overseas Base periodically with news about the fate of orders received. It is not difficult to design techniques whereby this can be accomplished cheaply and without overburdening the Overseas Base (which may be a supply ship) with irrelevancies. The advantage for the Base would be, for example, that temporary action could often be taken to alleviate a hardship where, without the information, there might be no justification for substitution, etc.

10. MINIMUM PROCESSING TIME AT THE SUPPLY CENTER

This quantity is of a very complex nature; it varies from one Supply Center to the other, depends on their state of activity, on the nature and quantity of the ordered item, the stock position, etc. Thus minimum processing time is defined as the (probable) minimum time needed to transform an order from its receipt into a ready package at the pier. To the minimum processing time must be added tidewater time which is the waiting time of the ready package at the pier until shipment.

The minimum processing time involves operations of two kinds: (a) within the Center, (b) outside the Center. It has therefore two components: first, the processing time proper which is the time the Center needs to identify, locate, pull out of storage, pack and deliver to pier the commodity ordered; second, a waiting time if an operation (b) is involved. This will be the case if the commodity is out of stock and has to be ordered from the manufacturer. Waiting time will only be positive if it exceeds the time it would have taken the Center to put

[9] We exclude the possibility that receipt of the item in advance of the date of the expected State of Deficiency has a negative value to the Overseas Base. This happens under certain circumstances, e.g., when storage difficulties prevail, unloading facilities do not yet exist, etc. Consideration of such circumstances can easily be incorporated in the model.

a properly packaged commodity which it has in stock - in the necessary quantity - at the pier. (It may be a short cut to have the manufacturer deliver directly to the pier ready for loading on the ship.)

The minimum processing time is not a rigidly given quantity even for the same commodity. It varies, for example, as a consequence of the expense one is willing to incur for handling; this can be viewed as a technological condition, determining a minimum accordingly.

Tidewater Time is the interval between delivery of the ready package at the pier and the sailing date (we have omitted loading time). It is composed of "free days" if the sailing date is the earliest one after date of receipt of the order which could be used allowing for minimal processing time but after the latter has lapsed (see Fig. 1). In the case of continuous transportation there are never "free days," because there is no waiting, hence no tidewater time at all (provided the capacity of the carrier corresponds to the flow of the materiel). Free days can be added by the Center to the minimal processing time of a particular order if this reduces costs and/or makes a speedier treatment of another order possible. Free days show that an earlier sailing date would possibly have been useful for the Overseas Base. When they occur it is an indication and measure of a shipping shortage and/or an unsatisfactory scheduling of shipping routes, which is not under the control of the Supply Center. Tidewater time may also be due to the Supply Center having missed the nearest sailing date - taking receipt of order and minimum processing time as criteria for determining it. Such tidewater time expresses a loss incurred by the Overseas Base as a consequence of inefficiency or congestion of the Supply Center. The loss can only be determined by measuring the importance for the Overseas Base of the delay in receiving the commodity. Finally, tidewater time may also have been chosen by the Supply Center in preference to later processing which would have brought the material to the pier closer to the sailing date (i.e., with less tidewater time). This might be the case if the working schedule of the Supply Center called for early removal of material from the warehouses, or because of early receipt from manufacturers, etc.

So we see that "tidewater time" is not a simple quantity. In fact, the several kinds distinguished above should be carefully held apart in statistical descriptions and in all evaluations of the operations of Supply Centers. However, it is safe to state that a system is working badly and is poorly designed when there is a great deal of tidewater time, whatever its reasons. Thus perhaps the greatest economies in present setups are possible by cutting tidewater time. According to the Pacific Requisitioning Guide (February, 1954) of the U.S. Navy, tidewater time often exceeds ocean shipping time quite considerably. While some of it is accounted for by loading and unloading, most of it can be attributed to the inner contradictions in planning which arise from observing deadline delivery dates and correspondingly neglecting sailing dates. A solution of the scheduling problem will bring substantial and instant[10] relief. Reforms may also be needed in the scheduling of ships and other carriers. Also in the Air Force ready cargo is often waiting much longer than it takes to airlift it to its destination. Similar observations hold for the Army.

---

[10]One ought to consider the employment of separate ships to remove the accumulated excess cargo simultaneously with the adoption of a consistent scheme of programming of the type outlined here. This effort would pay off in producing a marked improvement in the total level of supply overseas and produce a clear break in the two practices. It would not be very difficult to find this temporary, extra cargo space for this one big operation, which could be repeated from time to time as accumulations happen again.

In general, tidewater time is bound to be high if the efficiency of the Supply Center is good and the transportation system is of small capacity. A great amount of tidewater time may be due to various combinations of these factors by which it is produced.

11. FOLLOW-UPS

Every large supply system, military or commercial, will produce "follow-ups." These are inquiries by the Overseas Base to the Supply Center concerning earlier orders for which shipment has not yet been received. They aim to secure arrival of the goods. There are two kinds of follow-ups:

(a) Ordinary: These express the belief that the Supply Center, if it has received the correct order at all misses on one of these points:

(α) minimal processing time,

(β) sailing date,

(γ) type of transportation for the priority given to the order.

None are under the control of the Overseas Base, hence these follow-ups are operationally undesirable and lead only to confusion in the Supply Center. Inspection of performance there is desirable and necessary, but should be performed by others than an Overseas Base.

(b) Corrective: These change the nature of an outstanding order and can be made permissible in the system here described. (A full discussion of the conditions under which they may be allowed would lead too far at present.) These follow-ups essentially give new information to the Supply Center about an outstanding order. The information changes:

(α') the priority,

(β') the State of Deficiency date.

To be justified, apart from error, the corrective follow-ups must be due to changes in rates of activities that are supported by the Overseas Base, provided that the consequences of these changes can not be taken care of by standard local means (including the temporary compression of the consumption by the activities). Without this proviso there would merely be the occasion of a new order, which is always an alternative.

Each component of the entire supply system can arrange for follow-ups within itself. This is perfectly legitimate control of operations from an organizational standpoint. For example, the Supply Center can arrange to have the fate of incoming orders checked on a sampling basis at various stages of the processing. Indeed, such operations are necessary at least from time to time to secure efficient operation; but follow-ups of the type (a) from outside the Center are improper and confuse operations precisely where they are most easily disturbed, i.e., in the paper handling, issue departments.[11]

12. PEACETIME AND WARTIME LEVELS

A military logistic system is vitally different from civilian systems in that it may be expected to undergo violent expansion and/or suffer tremendous damage, both practically without warning. We discuss expansion first. The supply setup will be satisfactory only if it does not have to be changed in fundamental aspects (e.g., direction of flow of information, authority, ordering, etc.) when the expansion occurs, although obviously bottlenecks and other

---

[11] Cf. O. Morgenstern, "Note on the Role of Follow-Ups in the Naval Supply System," Progress Report, Logistics Research Division, U.S. Naval Supply Research and Development Facility, Bayonne, New Jersey, December 1952.

troubles have to be expected. It is, therefore, of primary importance to design a peacetime system so that it will withstand a great deal of pressure of this kind. For example, it would be bad if a high degree of mechanization would be of a capacity just taking care comfortably of peacetime levels and be able to cope with gradual expansions. A less perfect mechanization may be more elastic and also less vulnerable to physical destruction. Logistic thinking at present does not seem to consider these aspects sufficiently. There are, of course, grave problems here which are difficult to solve, but it is necessary to be reminded of their nature in any design of supply systems. It is easy, for example, to see that Deadline Delivery Dates of the current type are totally meaningless operationally once a modern war situation is encountered. They would lead to disastrous confusion were they then incorporated into decision-making. On the other hand, other criteria may stand the test of being logically acceptable although the volume of operations may pose serious difficulties. Yet these would occur within a structure that is basically sound.

It would lead too far to pursue these matters further here. The reader familiar with such situations will recognize the difficult nature of the processes alluded to (i.e., discontinuous type Markov processes, i.e., where the system is apt to undergo a sudden change into another state). Changes of the system need not be of a simple expansionary nature because the expansion is accompanied with the appearance of shortages relative to the new demands made upon the system. This introduces the question whether selections of essential demands from among all demands are possible; in other words, whether "compressibility" of the organization exists.

Next, a few words have to be added about vulnerability and damage of the system. A military supply system, as distinct from a commercial one, ought to be designed with a view to minimizing expected damage. Where its vulnerability lies depends on the type of attacks that may be directed against it. A system that may be attacked only on the fringes—e.g., cargo vessels sunk by submarines—is in a very different position from one where atomic attack may destroy a Supply Center, or several depots at once. Then the problem of dispersion may have to be taken into account, or underground installation, etc. Present logistical thinking has so far bypassed these possibilities. For example, it would be necessary to form ideas about the properties the system would have to possess so that it may be able to break into several self-contained parts with only difficult communication between them. Such a system may well look quite different from any presently known. Studies along these lines are urgently needed and, as a consequence, great changes may be indicated in the supply systems the three Services maintain at present. Outlines of alternative command setups are not sufficient and need not even be considered before the underlying economic structure has been adequately determined.

13. POLICY COMMENTS

The analysis has shown that while there are many possible supply systems only those that are free of inner contradictions are of importance. Whether a given system is free of these is not easily determined. In fact, existing systems have been known to survive for a long time even though they finally begin to exhibit such undesirable properties. Once they are discovered, however, an effort should be made to rearrange the flow of information, the line-up of authority and to reinterpret the mission of each component part. The scheme described in the preceding pages illustrates how this can be done. It would not be difficult to work out the necessary administrative details.

The determination of a feasible setup does not imply that it is optimal. But to discover the latter is at the present state of knowledge of logistic organizations an impossible task, especially in view of the required degree of complexity. Not quite so hopeless is the endeavor to discover, for a given, feasible structure those operations which tend to improve, and those which tend to impair, general performance. Knowledge about soundness of the structure combined with these indications is apparently all that can be hoped for at the present time.

Determination of performance requires measurements of the operations which in the military supply system cannot be given exhaustively in monetary terms. Thus the need to collect those statistics that are operationally meaningful becomes a major task. These statistics are only in fortunate, rare cases the same as those that are automatically obtained by collecting figures produced as by-products of functions carried out by the Supply Center, the bases, etc. Only by setting up a model describing the supply system in a logically satisfactory manner can one obtain the corresponding numbers that matter. This is a major task of any logistical investigation. It shall be discussed at another occasion.

* * *

THE CARGO-HANDLING SYSTEM[1]

R. R. O'Neill and J. K. Weinstock
University of California, Los Angeles

The study of the movement of goods, in the broad sense, must take into account all of the activities which occur between the producer of a commodity and the consumer. These activities are the basis of a complex transportation system; however, the complex system may be treated as a series of distinct components. The present research on cargo handling has been concerned with only one of these components, the movement of commodities between the land carrier and the ship.

The factors that make up the cargo handling system have been grouped into a limited number of independent elements: (1) the facility, (2) the inland transport, (3) the sea transport, (4) the commodity, and (5) the control. The sixth element, the process, is dependent upon the others and is the path (the coordinates in space and time) of the transporting agent and the commodity. These elements are represented pictorially in Figure 1.

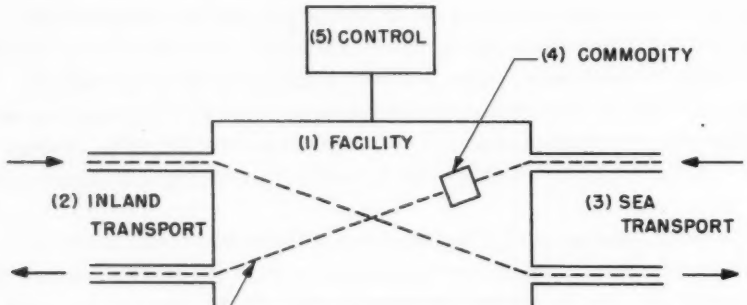


Figure 1. The cargo-handling system

It is postulated that these six major elements completely define the cargo-handling system. Thus a knowledge of these elements is both necessary and sufficient to the understanding of the transfer of cargo between a land carrier and a ship. However, the components of each of the major elements and their interactions are difficult to quantify. As a consequence, specific activities within a facility were inspected. These activities were viewed as a series of cyclic operations where each prime mover or transporting agent carried a commodity from one point to another, and then returned to repeat the operation. A schematic diagram of a typical loading sequence is shown in Figure 2.

---

[1]The research for this article was done under Office of Naval Research Contract Nonr 233(07). This article is an abridged account of "An Engineering Analysis of Cargo Handling," Department of Engineering, University of California, Los Angeles, to appear in April 1955. Acknowledgement is given to all of the other members of the staff for their contributions to the material which is presented.

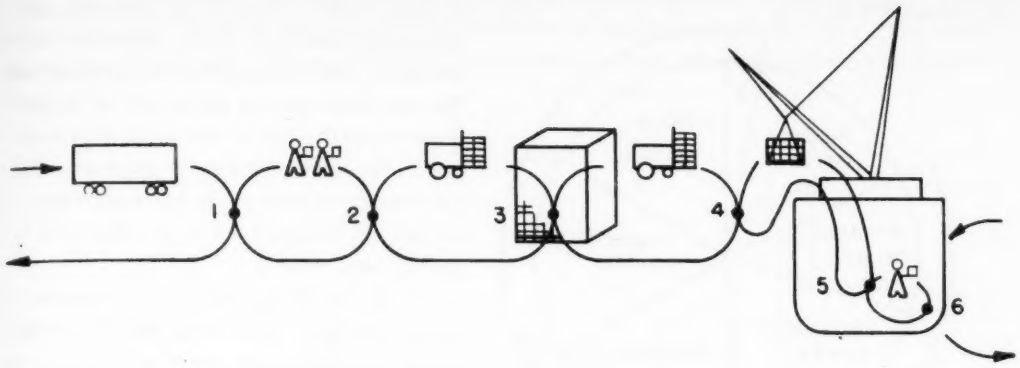


Figure 2. Cargo handling as a series of cyclic operations

The commodity arrives at point 1 by rail. It is unloaded, one carton at a time, to a pallet at point 2. A fork-lift truck picks up the pallet load and moves it to a position in the transit shed, 3. After a period of time in storage the commodity is moved by fork-lift to the apron, 4. The pallet is hoisted by the hook over the side of the ship and into the hold, 5, where the pallet is unloaded by the hold men, one carton at a time, and stowed in the wings, 6. Each of the separate operations is called a link. The term cycle has been reserved for a single performance of a link by a transporting agent. The various activities which can occur during a cycle have been grouped into four categories. These are pick-up (P), the transport loaded (L), the release (R), and the transport empty (E). In addition to the productive activities, a cycle may also contain delays. A delay that results when a transporting agent is not able to pick-up or set down a load because one of the connecting links is not in phase is called an induced delay (D). Any other delay such as a breakdown or a stoppage within a link is called an internal delay (A). A complete cycle or the total time of a cycle for any link can be represented by a pie chart as in Figure 3.

The sum of the productive times plus the internal delays, which are inherent in a cycle, is called the operating time. In addition to the primary activity times noted above, there are miscellaneous activities making up a relatively small percentage of the total time.

The activity times represented by Figure 3 can be values for a single cycle or an average of many repetitive cycles; for example, the average value of the total cycle time is called the mean cycle time, the average value of pick-up time is called the mean pick-up time, and so forth.

The connecting points between links are called nodes. It is at the nodes that a commodity can be stored for a period of time or transferred directly to another transporting agent. It is also at the nodes that the characteristics of a commodity can be changed. For example, the commodity may enter as a single carton and leave as a load of 36 cartons on a pallet.

The rate at which the commodity is moved from one node to the next is defined as the weight or volume of the commodity carried by one transporting agent per trip, times the number of transporting agents in the link, divided by the average time required to make one trip. If during an interval of time, the net change in the amount of storage at a node is zero, the

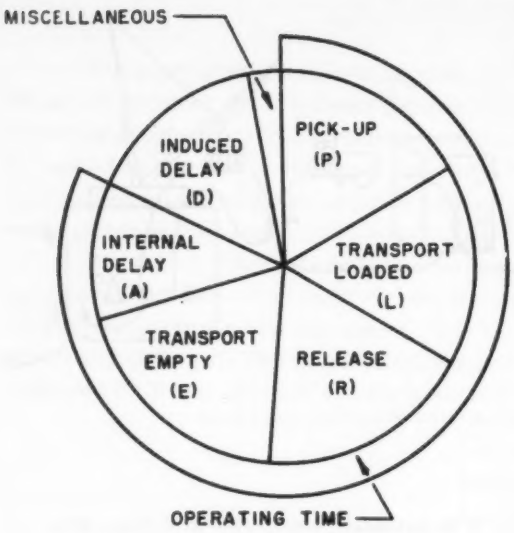


Figure 3. Total time of a cycle

average rates of flow in the preceding and succeeding links are equal. However, the operating rate of each link may be different. The operating rate is the weight or volume transported divided by the mean operating time. The operating rate is determined by the characteristics of the particular link only and is assumed not to be influenced by the adjacent links.

At the gross level a port terminal can be thought of as a node; with a land carrier as the link on one side and a ship as the link of the other side. In this way it is seen that the link-node concept may be used in a more generalized transportation system. However, for the purposes of this study, a limited number of operations within a terminal are considered. They are the links that connect the point of storage on the wharf and the point of stowage in the hold of a ship. These links have certain general characteristics in common, for example, each is the round-trip path of a transporting agent. In their more specific characteristics, however, the links may be quite dissimilar. The links that were actually studied are the following:

The Hook Link. This is the link which is easiest to identify, and the other links are usually traced from it. The transporting agent in the hook link is the combination of the hook, cables, booms, winches, and the operators of the equipment. This link is illustrated in Figure 4, which is representative of West Coast practices.

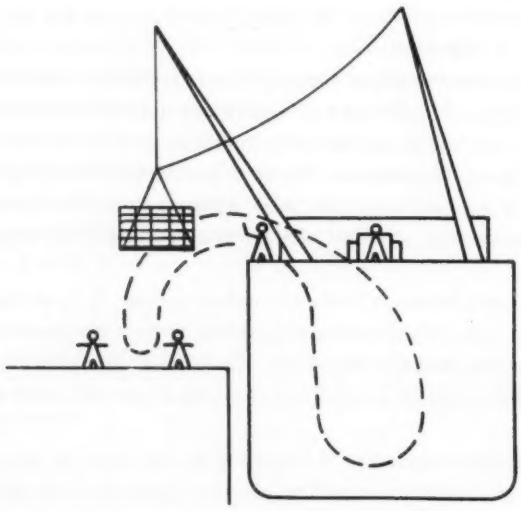


Figure 4. The hook link

The Wharf Link. The operation in this link is the movement of a load between a point on the pier or quay, directly below the hook, and a point in the shed or storage area. The transporting agent is usually a tractor-trailer combination or a fork-lift truck and the driver (see Figure 5).

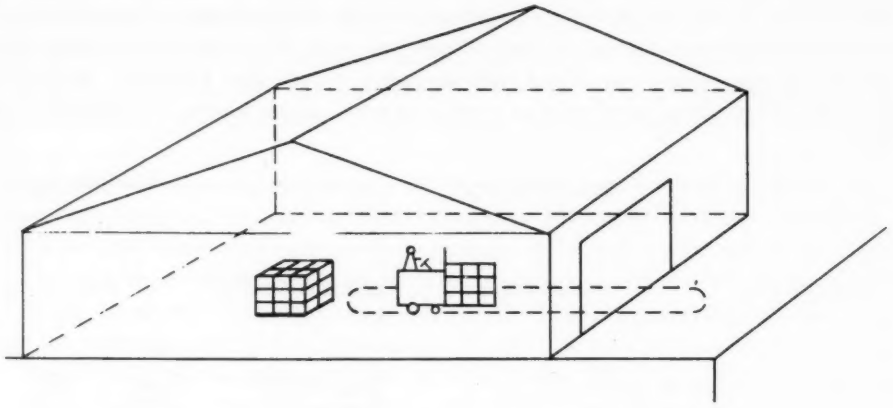


Figure 5. The wharf link

The Shed Link. This link connects the wharf link to the final point of rest in the transit shed or outside storage area. The transporting agent may be a fork-lift truck with driver or a crew of longshoremen who are loading a pallet which will in turn be picked up by a fork-lift truck in the wharf link (see Figure 6). The shed link may not exist in all cargo-handling operations. For example, if a cargo is palletized by the supplier, all that is necessary is a fork-lift truck that can move the load from the storage point in the shed to the point below the hook (i.e. wharf link).

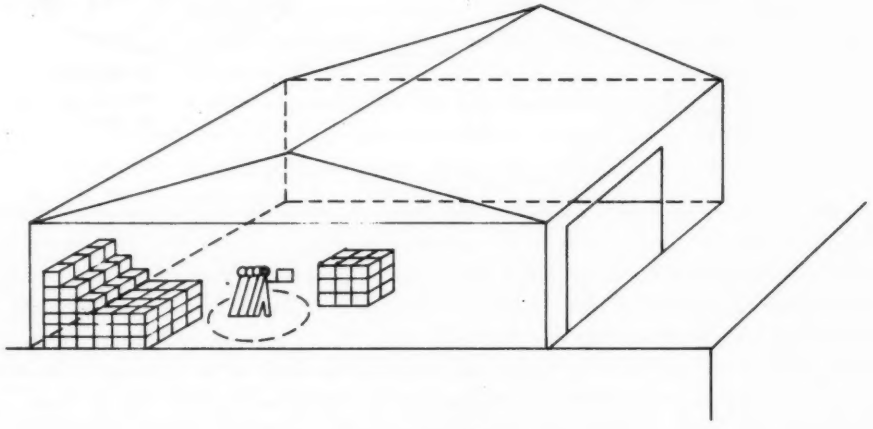


Figure 6. The shed link

The Hold Intermediate Link. In this link the load is moved between a point beneath the hook in the square of the hatch and some point in the wings. The transporting agent may be one

or more men in combination with a hand trailer or a fork-lift truck (see Figure 7). This link may exist during loading operations but it rarely exists during unloading. In many unloading operations the hook drags the cargo from the wings into the square of the hatch and then proceeds to lift the cargo out of the hold.

The Hold Link. This includes the movement of a load between either a point directly beneath the hook or the end of the hold intermediate link and the stowage position in the hold. The transporting agent is usually a group of men who may, for example, be unloading a pallet and placing the commodities into their final stowage positions (see Figure 8). In some specific instances the transporting agent may be a fork-lift truck which moves a unitized load into position.

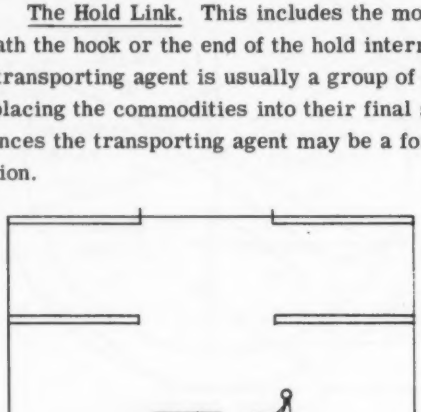


Figure 7. The hold intermediate link

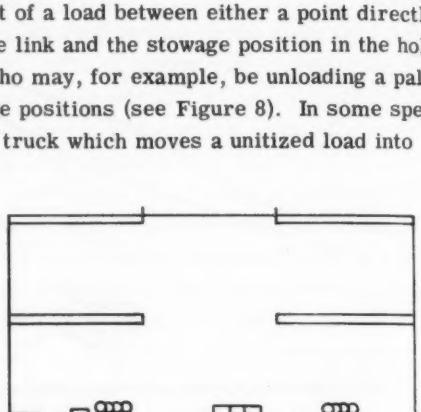


Figure 8. The hold link

Schematically then, the links which were observed fit together as shown in Figure 9. The shed and hold intermediate links are shown as dotted lines because both or either of them may or may not occur during any particular cargo-handling operation.

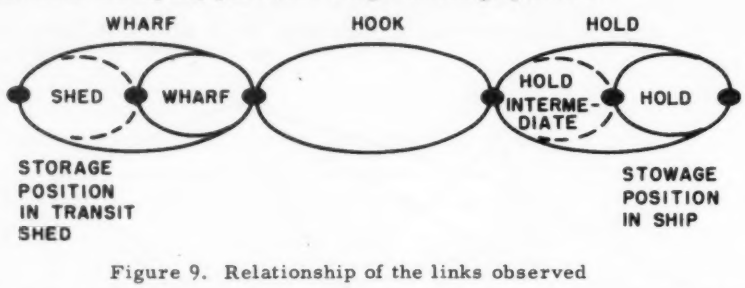


Figure 9. Relationship of the links observed

When a more general view of the link-node arrangement is considered, it is evident, as noted earlier, that the rate of flow of the commodity through each of the links is equal. However, the operating rate for each of the links may be different. This difference is caused by the links being unbalanced in their ability to keep the commodity flowing and is made up of the induced delay, i.e., the time that one link must wait for its connecting links to catch up. From the foregoing it is evident that an improvement could be made to the system by establishing a better balance between links. The first approach is to find the slowest link in terms of the operating time and to increase its rate of movement in line with the others. This inter-link concept of the cargo-handling system, which is almost axiomatic, has been written into an hypothesis, as follows:

> Hypothesis I: If there is a group of transporting links connected in series, then there is one link which has the least mean induced delay and is the slowest or controlling link.

The question now arises as to what can be done to the controlling link to improve its performance? First of all, it is necessary to point out that the performance times for the different activities of a link are relatively independent. For example, a fork-lift truck that moves a load 500 feet would have a different mean transport loaded time than a truck which moves a similar load only 50 feet, even though the mean pick-up time would be about the same in both cases. Because of this independence, the effect of the different variables has to be related to each of the activities separately. A knowledge of these variables and their interactions will then enable decisions to be made which will improve the particular activity and in turn the performance of the link. In hypothesis form the foregoing intra-link relationship can be stated as follows:

Hypothesis II: The relationship between the relevant factors which affect the mean activity time of a link can be formulated so as to make prediction possible.

Both of these hypotheses were used as the basis for a field study made in the summer of 1953. The data which would provide the maximum amount of usable information to test the hypotheses would consist of a complete time history of the handling operations. The usual techniques for gathering this kind of information are those of conventional motion and time studies. For several reasons, recourse was made to another technique which provided less but adequate information. The technique employed was one sometimes used in worker productivity studies and is called work sampling. Specifically, time instants were chosen at random beforehand. At each of the instants chosen, the operation being performed was observed, and the link element in progress noted. The percentage of time required by each particular operation of the work cycle was to be estimated by the percentage of observations made when that cycle element was in progress. For example, if 100 observations were made at random and the link was in a pick-up state during 20 of the observations, then the estimate of the pick-up time is twenty percent.

The observations were made at random times so that the tools of statistical inference could be used to establish confidence intervals around the estimates. A tally was also kept of the number of trips made by the transporting agent. From the tally data, the mean cycle time was calculated. In addition, all of the physical characteristics of the system which seemed to be pertinent were measured and noted.

In all, eighty-two studies were made in the Los Angeles and San Francisco areas involving 41,218 individual observations. Each study encompassed a period of two to three hours during which the major factors remained constant. The data collected were then reduced and analyzed with reference to Hypotheses I and II.

A test of Hypothesis I was made by comparing the percentage of time each of the three major links was in induced delay (see the first row of Table I). The shed and hold intermediate links are not included because observations on them were made in relatively few of the studies and because in general these links have been found to have subordinate roles. The data used in the comparison are the pooled values obtained from all the studies of loading and unloading operations in both the Los Angeles and San Francisco areas.

Although neither the weight nor the volume appear explicitly in the determination of the mean operating time or mean induced delay, both were taken into consideration. For each loading and unloading operation, the average rate of flow was the same for all the links between the points of rest on the wharf and in the ship. In most cases, data were obtained simultaneously for all of the active links; thus for each study the mean activity times of the links may be compared directly.

TABLE I

COMPARISON OF THE THREE MAJOR LINKS TO DETERMINE THE SLOWEST LINK

	Wharf	Hook	Hold
Pooled Value of Induced Delay (Percent of Time)	39.2	29.0	40.6
Upper and Lower Quartile Points[2] of the Mean Induced Delay (Percent of Time)	22-54	18-37	30-54
Slowest Link (Percent of Studies)	19	53	28

From the comparison of the pooled induced delay values in Table I it is seen that the hook was, on an overall average, the slowest link. However, the values of the mean induced delay obtained for each of the studies of the hook are highly variable. For example, it is noted that fifty percent of the weighted studies of the hook fell within the range from 18-37 percent. This suggests that the hook was not the slowest link in all studies. In the 39 studies where data were obtained simultaneously for all active links, a direct comparison indicates that the hook controlled in more than half (53 percent) of the loading and unloading operations.

The finding of the hook link as the controlling link in more than half the studies may be unexpected. However, it is not unreasonable if consideration is given to the fact that the hook link, with its single transporting agent, is the most inflexible link in the system. Through experience the number of transporting agents used in the more flexible links would be expected to be chosen so that these links could equal or surpass the performance of the hook link.

Another significant result obtained from the comparison of the induced delays is that the overall average delay in even the slowest link is still very high (29.0 percent). A possible explanation is that it is caused by the cycle-to-cycle variation in the operating time of a link. Thus, if three links in series each have operating times that vary from cycle to cycle, then a different link may be the slowest during each cycle. This in turn may cause an induced delay in the next cycle of the two faster links. The result will be an appreciable mean induced delay in all links.

A complete analysis of the effects of cycle-to-cycle variability is one of the current problems being worked on by the project.

The testing of the data with reference to Hypothesis II was not as conclusive. In general the positive results obtained were as anticipated. For example, the sum of the transport loaded and empty time for the wharf link was correlated with the distance that the transporting agent moves. However, it was not correlated with the weight or volume of the commodity. Another example, is that the operating time of the hold link was found to be correlated with the weight and volume of the commodity.

The analysis above is a partial evaluation of the present day cargo-handling system. As a further step, the same idealization presented could be used as a basis for the design and evaluation of radical systems in which the present restrictions are removed.

* * *

[2]The upper and lower quartile points were determined by ranking the values of the mean induced delay time, weighted according to the numbers of observations in each study, and selecting the middle 50% range such that 25% of the values were greater than the upper point and 25% less than the lower point.

A SCIENTIFIC METHOD FOR ESTABLISHING REORDER POINTS

Morris J. Solomon[1]
U. S. Air Force

This paper deals with the inventory problems of an operating installation such as an air base, a camp, post or station. The method described will be in terms of an Air Force base, but it applies equally to any installation with a clearly defined objective. The discussion will be nonmathematical. Those interested in a more detailed description of the proposed procedure are referred to a more technical article of the author [1].

INVENTORY PROBLEM OF AN AIR BASE

An air base has a certain number of aircraft in operation. Repairs are made on these aircraft as they become necessary. If a part is needed before the plane can resume operation, it is classified as a vital part. We shall be concerned only with vital parts. Ordinarily, an airplane contains thousands of vital parts. When a vital part is needed and is not available, the aircraft will be grounded for as long as it takes to receive the part from a central depot on a rush requisition. To keep down such delays a base keeps a stock of parts on hand and maintains a reorder point, which is the level of on-hand and on-order quantity of a part that is the signal for a new order.

The two main problems of the base are how many of each part to order, and when to order them. The problem of how much to order has a standard answer which can be found in textbooks on business management [2]. This answer, with some modification should be satisfactory for the base. Until recently, the problem of when to reorder (reorder point) was left to judgment. Recent work [3] has provided a guide for setting reorder points. In all such work however, each part or item is considered as leading an independent existence. What the base needs is a method which treats the flying of the plane as a chain which is no stronger than the weakest link, the links corresponding to spare parts. The situation has been well expressed in an unofficial Air Force description of a weapon system.

"All of us remember the old legend - how for want of a horseshoe nail a kingdom was lost. Nail, shoe, horse, rider, and blacksmith, constituted in those days a weapon system in which each part was dependent on all the others. In the broadest sense of the word, we may compare the horseshoe nail with any part of an airplane - a generator, one of the many black boxes we all know about - or, if we wish to carry it to an extreme, to a nut, or a bolt which holds some vital part in place, and without which the aircraft, as a weapon system, fails."

RISK OF BEING GROUNDED AND MINIMUM ROUTINE SUPPLY COST

Any set of reorder points for the various vital parts involves some risk of a plane being grounded in an hour of operation because the base lacks one or more parts. In general as the reorder points are made larger this risk becomes smaller and the routine supply cost becomes greater. This is true for any system for setting reorder points. The first step is to be able to set reorder points for all vital parts for different risks so that the routine supply

[1]Now with Bureau of the Census.

cost is a minimum. By routine supply cost is meant the cost of parts, storage, obsolescence, routine paperwork and some meaningful interest on investment. Stated otherwise, our initial aim is to be able to set reorder points so that for a given routine supply cost, the base runs the least risk of grounding planes because it lacks parts. This phase of the problem is illustrated in Figure 1.

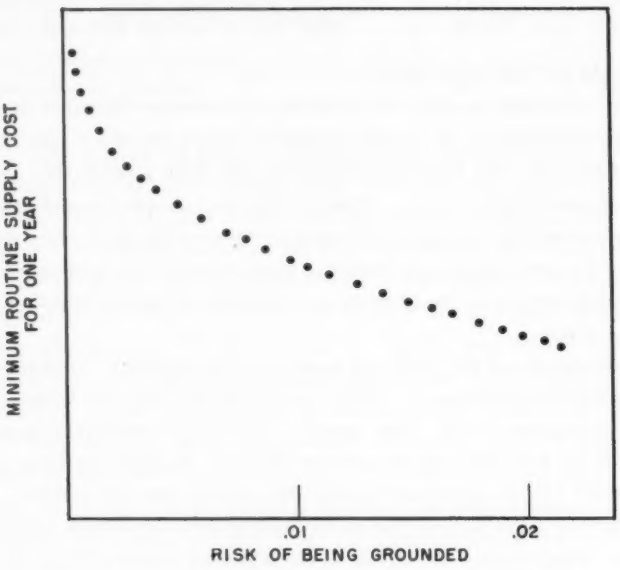


Figure 1. Risk of being grounded and minimum routine supply cost

The risk of being grounded and the minimum routine supply cost depends on the following factors:

1. The average consumption of the part during the time it takes to get delivery
2. The uncertainty of the average consumption
3. The cost of keeping a part in the base inventory
   a. Base storage cost
   b. Interest and obsolescence
   c. Cost of processing a requisition
4. The order quantity
5. The frequency of needing a part
6. The chain effect

Each of these factors has a direct bearing on the risk or the routine supply cost. The average consumption affects both. Less obvious, but nevertheless important is the uncertainty that surrounds the average consumption. In fact, it is the uncertainty which creates our problem. If we knew exactly how many parts would be required in each period, we would have that exact amount as our reorder point and there would be no problem. The greater the cost of keeping a part in the base inventory, the smaller we would expect the reorder point to be. The quantity of a part that is ordered affects the risk of being grounded. If the quantity ordered of a part is sufficient for several years, there would be very little risk of running out of that part. The frequency of requiring a part is related to the average consumption, but it is

not identical to it. The "once in a blue moon" part can be requisitioned as needed if the requirement is infrequent enough. The chain effect is the dependence of the activity of flying a plane on having each of every spare part when required.

It is possible to express each of these factors mathematically in a meaningful equation as done in [1]. Then, using maximizing techniques from sampling theory, one can solve for the "best" reorder points. By "best" for the time being, we mean the reorder points which would give us the least risk for a given routine supply cost or the least routine supply cost for a given risk. By doing this for successive costs we obtain a series of points such as are plotted in Figure 1.

But Figure 1 is only the first step. Which risk of being grounded (and reorder points) should the base choose. Factors bearing on the risk to be chosen are:

1. The minimum routine supply cost (depending on risk)
2. The delay in getting a part on rush requisition
3. Cost of rush requisitions
   a. Unit cost of processing
   b. Frequency (depending on risk)
4. Cost of extra aircraft required to make up for grounding
   a. Annual cost of having an extra aircraft in the inventory
   b. Number of extra aircraft required due to grounding (depending on risk)

It can be seen that there are conflicting forces at work. A low risk is contrasted with a high risk below:

Low Risk	High Risk
Large Routine Supply Cost	Small Routine Supply Cost
Small Rush Requisition Cost	Large Rush Requisition Cost
Little Lost Aircraft Time	Much Lost Aircraft Time
Small Cost of Extra Aircraft	Large Cost of Extra Aircraft

These forces can be expressed mathematically in an equation yielding the overall cost per flying hour. We can expect that the relationship would be bowl shaped similar to the curve drawn in Figure 2.

Figure 2 can tell us which risk is the least costly in an overall sense. For the moment let us assume that we choose the least cost risk. Corresponding to this point there is a set of reorder points (one reorder point for each part).

So far our method has a serious drawback. It involves a complicated computation which is good for one instant in time. Each time there is a change it would appear that another computation would have to be started. Fortunately, once results are obtained, they can be put in the form of a table which is both easy to use and satisfactory under changing conditions. The results can be put in the form of Table 1. In it the reorder point is given as a multiple of the average requirement in the order to delivery period. A base supply clerk can derive the reorder point for any part from this table. He looks up the value in the table corresponding to the individual part's average requirement and its cost factor (see [1] pp. 424-426), and multiplies the average requirement by this value. For example if the average requirement for the part is 16 and the cost factor is 7.0, the value in Table 1 is 1.2. The reorder point would be 1.2 x 16, or approximately 19.

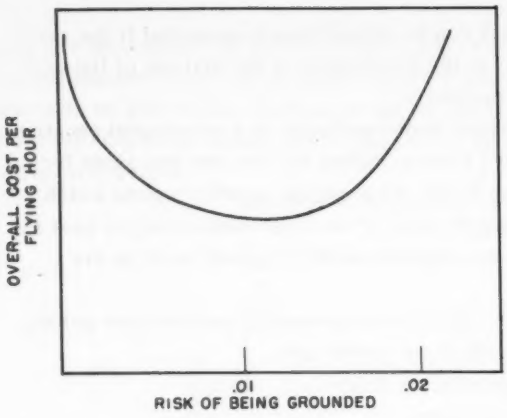


Figure 2. Risk of being grounded in an hour of operation and over-all cost per flying hour

The entries in Table 1 are fictitious but are given here to illustrate the format of such a table. We would expect that the greater the cost factor, the smaller the reorder point should be. The larger the average requirement, the smaller the multiple in the table. This is because we can expect a small average requirement to vary more widely on a proportionate basis.

The factors which our method takes into account have been recognized as important by supply people at various times. But the recognition is partial and is not incorporated systematically in a mathematical model that can be checked and implemented. For example, two articles in the June 1954 issue of this publication [4] listed many of the factors and some of the relationships I have enumerated. Rear Admiral F. L. Hetter in his article "Navy Stratification and Fractionization for Improvement of Inventory Management" mentions frequency of demand and cost of item (related to cost of carrying a part). Captain Charles F. Lynch in his article "Applied Logistics In The Navy" discusses the factors of uncertainty of demand, cost of carrying a part in the inventory including interest and obsolescence. In addition he is interested in "standards for stock levels that will meet practical requirements, i.e. that can be executed easily in the field and yet adhere adequately to the requirement found by analysis."

One military agency claims great gains from categorizing parts by frequency of demand. Another military agency sets reorder points on the basis of the cost of the part.

The writer believes that the ideal destiny of an optimum solution is its embodiment in simple rules to run a system in a near optimum manner. The would be optimizer should constantly look for rules of thumb which will be almost as good as the theoretical optimum - without the administrative disadvantages of the latter. If a simple system skims off enough of the available cream, it may be preferable on an overall basis to a more complicated system. But at the very least, a more complete model can be used as a yardstick by which simpler set-ups can be judged.

DYNAMICS OF THE PROBLEM

Up to now, the discussion has been in terms of an Air Force that is starting from scratch. At certain points in time this may be fairly realistic. But most of the time actions have been taken in the past which have to be lived with, and have to be used as a point of departure. An important principle in this regard is that the only costs which are relevant for decision making are costs which the organization can avoid incurring in the future. This means that costs change with time. What is a controllable cost before the event is water over the dam after the commitment is made. For example, if the Air Force has over-procured badly on some part so that no further procurement is likely, when this becomes evident, certain costs, such as those for obsolescence and interest on capital become zero or close to zero. The form of Table 1 is such that it can handle changed cost factors or other factors, and it is believed that the optimum solution will not be sensitive to a moderate amount of change even if changes

TABLE 1 - Optimum Reorder Points of Vital Aircraft Parts of the T-48 Aircraft as a Multiple of the Average Requirement (entries are fictitious)

Average Requirement	Cost Factor of Part						
	Less than .26	.26-1.00	1.01-5.00	5.01-10.00	10.01-50.00	50.01-100.00	Over 100
Less than .05	0	0	0	0	0	0	0
.05- .20	10.0	0	0	0	0	0	0
.21- .50	6.0	5.0	4.0	3.0	2.0	1.0	0
.51- 1.00	5.0	4.0	3.0	2.0	1.8	1.0	1.0
1.01- 2.00	4.5	3.8	2.7	1.8	1.0	1.0	1.0
2.01- 5.00	4.0	3.2	2.3	1.5	1.0	1.0	1.0
5.01- 10.00	3.5	2.8	2.0	1.3	1.0	1.0	1.0
10.01- 20.00	3.0	2.3	1.7	1.2	1.0	1.0	1.0
20.01- 50.00	2.5	2.0	1.4	1.1	1.0	1.0	1.0
50.01- 80.00	2.0	1.7	1.2	1.0	1.0	1.0	1.0
80.01- 100.00	1.5	1.4	1.1	1.0	1.0	1.0	1.0
Over 100.00	1.3	1.2	1.1	1.0	1.0	1.0	1.0

are in one direction. The sensitivity of a solution is something which should be tested in practical situations.

Another type of change will tend to require a new Table 1. After aircraft have been purchased, the factor of the amortized cost of extra aircraft should not be considered.

The relation of the base to the rest of the organization is important. The base is not in business for itself. To reflect this, the cost factors must be computed from the point of view of the Air Force as a whole rather than the base.

Up to now we have talked about money costs only. There are situations where the importance of the mission requires a small risk of being grounded. In such a case, the risk can be set directly. Our method tells us which are the cheapest reorder points for the risk we have chosen.

A military organization must plan to operate with one eye on the peacetime dollar cost and the other on what mobilization potential will be available. It may choose a higher cost per flying hour (on a peacetime basis) in order to have a more satisfactory mobilization base. The difference in cost between the minimum cost per flying hour and the one finally chosen is in the nature of an insurance premium which the organization chooses to incur. Similarly, a higher cost per flying hour may be chosen for a strategic advantage such as less dependence on a central depot. To the extent that the curve in Figure 2 has a shallow bottom, nonmonetary considerations can be taken into account at relatively little cost.

Basically our method does no more than what a sharp supply man would try to do, but it uses the latter's common sense in a systematic way. The net effect is that many considerations and their interactions can be handled at the same time.

REFERENCES

[1] Solomon, Morris J., "Optimum Operation of a Complex Activity Under Conditions of Uncertainty," Journal of the Operations Research Society of America, Vol 2 (1954), pp. 419-432.

[2] Koepke, Charles A., Plant Production Control, 2nd Edition, John Wiley and Sons, 1949.

[3] Laderman, J., Littauer, S. B., and Weiss, Lionel, "The Inventory Problem," Journal of the American Statistical Association, Vol 48 (1953), pp. 717-732.

[4] Naval Research Logistics Quarterly, Vol 1 (1954).

Rear Admiral F. L. Hetter, "Navy Stratification and Fractionization for Improvement of Inventory Management", pp. 75-78.

Captain Charles F. Lynch, "Notes on Applied Analytical Logistics in the Navy", pp. 90-102.

* * *

THE THEORY OF INVENTORY MANAGEMENT—A REVIEW[1]

Thomas V. V. Atwater, Jr.
Massachusetts Institute of Technology

This article is a review of the book, The Theory of Inventory Management, by Thomson M. Whitin. Princeton University Press, Princeton, New Jersey, 1953, 245 pages.

The Theory of Inventory Management, by T. M. Whitin, now Assistant Professor of Industrial Management at Massachusetts Institute of Technology's School of Industrial Management, is in many respects a remarkable book. For one thing, it represents valuable evidence of the utility of government-sponsored, university-directed research in areas in which a genuine matching of interests can be discovered. This book first materialized in the form of a doctoral dissertation supported by ONR.[2] The author's consequent dual motivation is clearly reflected in the plan and content of the book, which make up a second point of remarkability.

Whitin's book has already enjoyed a respectable measure of success with an unusually diverse audience. Economists with interests ranging from the purely abstract to the purely measurable, operating businessmen and business consultants, teachers of business subjects, members of the armed forces, operations research specialists, mathematicians, and statisticians do not often find a book which provides so substantial an overlap of interests. Because Whitin is a talented and well-trained economist, the promise of the title is fulfilled, and The Theory . . . is indeed one . . . of Inventory Management. That is, the book is primarily devoted to operational application. The techniques of economic analysis are turned on a specific (though admittedly extremely complex) operating problem area. The principal techniques used are marginal analysis (which utilizes the calculus), game theory, simple probability analysis; and the collection, analysis, and presentation of relevant empirical data, both primary and secondary. However, a reader with practical experience and responsibility for stock control can profit from the book without training in any of the formal methods used, if he will take certain conclusions on faith.

This latter point is the final general aspect of the book I wish to go into here. Application of formal analytical methods to complex practical problems is not easy; it is in fact a demanding art which requires high-level analytical skill, disciplined judgment, ingenuity and hard work. It is sometimes even more difficult to translate and illustrate theoretical results for the benefit of untrained readers. Whitin does this, and also provides a non-technical explanation of the logic of his methods, in the first, most general, section of his book. I do not wish to imply that readers with no training in economics or mathematics will find The Theory . . . light going. Whitin's style is spare and precise. The last two sections of the book are probably too specialized for the general reader, but he has been courteously treated in the four main chapters.

---

[1]Page references not otherwise identified in the text of this paper are to this book.

[2]Prepared under contract N6 onr 27009 with the Office of Naval Research; submitted to the Department of Economics, Princeton University as a doctoral dissertation written with the advisory direction of Professor Oskar Morgenstern.

CONTENT: OUTLINE

The Theory of Inventory Management is divided into an introductory Chapter, which reviews the problem generally and establishes the quantitative importance of inventories in our economy, and three sections which, as I have mentioned, reflect dual but parallel interests.

Part I, entitled "Inventory Levels for the Firm," contains the theoretical core of the book, as well as the most detailed illustrations and Whitin's original contributions to knowledge in this area.

Part II, "Inventories in the Economy," is made up of two quite different chapters. These are by no means digressions, but they can be read separately, in reverse order, or omitted by those whose chief interest is at an operating level. For this reason, I will confine my comments on this part of the book to this paragraph. Chapter 5, "Inventories and Business Cycles," reviews the treatment accorded inventories by several business cycle theorists, and also the findings of empirical researchers. With the findings of Part I as evaluative criteria, Whitin has produced a short essay of considerable interest to students of the business cycle. Chapter 6, "The Role of Inventories in General Equilibrium Models," covers four particular models, including the three which have been specifically designed for quantitative application—those of Leontief[3] and Wood and Dantzig.[4] This chapter, devoted largely to the problems of building realistic stock and flow relationships into the models, is chiefly for the econometrician. The exposition in both chapters is clear enough for the nonspecialist to get the feel of the subject matter, but this is hardly beginning-text material. On the other hand, the specialist will note that Whitin has by no means exhausted the cyclical implications of his more general findings.

Part III, "Inventories in the National Military Establishment," drops in Chapters 7 and 8 to lower levels of analysis. These chapters are devoted to general problems encountered in military applications of the theory of Part I, (Chapter 7) and with evaluation of Naval inventory control processes (Chapter 8). Chapter 9, "The Problem of Military Value," shoots back up the scale of abstraction, providing illustrative examples of the game theory approach to the evaluation of strategies. To this reviewer, Whitin overstates, in this section, an aspect of the inventory problem which he had by contrast underemphasized in Part I. The following sections, devoted to some of the finer points of Parts I and III, will raise this "value" question after a brief exposition of the theoretical framework.

THE THEORY

The relative ease with which Whitin's work can be applied in a real-life context follows from the fact that he has translated his work into operating terms. The basic models turn on the determination of economic purchase (or manufacturing lot) quantities and optimal safety allowances, but exposition is carried out in terms of the two basic types of inventory control system currently in use; the two-bin system, in which the timing of orders depends on experienced demand, and the ordering cycle system, in which the frequency of stock-level review, and probably but not necessarily the frequency of orders, are predetermined or scheduled.

Economic lot sizes or purchase quantities will be identical under these two types of system. Optimal purchase or manufacturing lots are determined by balancing costs which

---

[3]Leontief, W. W., The Structure of the American Economy, 1919-1939, New York: Oxford, 1951.

[4]Wood, Marshall, and Dantzig, George, "Programming of Interdependent Activities. . .," Econometrica, Vol. 17 (1949), pp. 193-211.

increase with lot size (carrying charges, including interest, storage, obsolescence, etc.) against costs which are inversely related to lot size (ordering or set-up costs, unit cost where quantity discounts are available, costs of being out of stock, and the like). These variables, and others, such as price expectations, which should influence lot sizes, are not systematically related to the method used to signal the need for replacement. If demand over time were known with certainty, the lot-size formulas would uniquely determine order frequency per period and also average stock levels and turnover.

In the simplest case, Whitin assumes:

> . . .(1) that the expense of procurement is a constant amount for each order placed and (2) that interest, risk, depreciation, obsolescence, etc., may be lumped into one percentage figure I. Let Y designate the expected yearly sales (in physical units). Let Q be the economic purchase quantity (in physical units), let C be the unit cost, and S be the procurement expense involved in making one order (in dollars). Then total annual variable costs (TVC) may be expressed as follows:

$$TVC = \frac{QC}{2} I + \frac{Y}{Q} S$$

> In order to determine the Q which minimizes total costs, we differentiate the above expression with respect to Q and set the derivative equal to zero. The following equation is obtained:

$$\frac{IC}{2} - \frac{YS}{Q^2} = 0$$

> which results in the solution:

$$Q = \sqrt{\frac{2YS}{IC}}$$

> This equation states that Q, the economic purchase quantity, varies directly with the square root of expected sales and the square root of procurement expenses and varies inversely with the square root of the carrying charges.[5]

Whitin further considers cases which take account of quantity discounts, price anticipations, and variable storage costs.

Since future demands are rarely known with certainty, it is necessary to add safety margins to the average levels determined in the above manner. The technique is similar, involving a balancing of the costs of depletion (lost sales, customers, good will), which are inversely related to quantities stocked, and carrying charges, which, as above, rise with a rise in stock levels. Whitin takes pains to point out that the inventory costs of a "never-out" policy can be very high, so that a rational choice among economically attainable levels of protection against stock-outs is involved.

At this point, by comparing the safety-allowance requirements of the two-bin and ordering cycle systems, Whitin's work provides a nice demonstration of the utility of economic analysis as an aid to policy formation. The ordering cycle systems require measurably larger

---

[5] T. M. Whitin, op. cit., pp. 32-33.

safety allowances than the two-bin systems because a scheduling constant is injected between the time-paths of realized demand and orders for replenishment. Since the two types of system call for identical lot or purchase quantities, and therefore for the same annual ordering frequency, the difference in safety margins (and therefore in average stock levels and carrying charges) must be offset by increased administrative or other efficiencies if use of the ordering cycle systems is to be justified.

THE STYLE-GOODS CASE AND THE VALUE PROBLEM

Whitin also considers (Part I) the interaction between lot sizes and safety margins, a matter which has not formerly been treated. A more important contribution is Whitin's development of a "style-goods" case which can serve, in fact, as a bridge between Parts I and III. The reason for this lies in the fact that, in the style-goods case, the "values," as well as the costs, associated with the stocking of a particular item move from the assumptions-framework of the problem into an explicit parametric role. In the most general model, selling prices are ignored, and demand in either physical units or dollars (at cost) is taken as known in probability terms: thus the generally important commercial relationship between volume demanded and selling price is removed from the problem. While this is a completely legitimate—and extremely useful—way to go at a particular aspect of inventory control, it nevertheless represents what Charles Hitch has called "sub-optimization,"[6] and the method has utility only where the price-volume relationship is for one reason or another (market conditions, firm price policy, etc.) irrelevant. It has long been commonplace practice, of course, to utilize price changes as a substitute for or supplement to inventory planning.

A style-goods problem could be defined, however, as being primarily one of having the right item on hand at precisely the right time, and this is clearly a definition which could be carried over to "the problem of military value." From the seller's point of view, the equivalence is extended by the fact that the ultimate determination of the "right item" rests in some measure in the hands of a relatively few key customers whose reactions must be anticipated, along with those of rival sellers. In spite of the fact, then, that business problems can all be cast in reasonably objective monetary terms, the "value" problem is not necessarily any simpler in business than it is in the case of the armed services. In Part III, Whitin suggests and briefly illustrates the game theory approach to this pervasive knot of questions. While it is clear that game theory has helped enormously in encouraging precise statements of such problems, and has also provided a substantial aid to insight and intuition, at the present time specific applications of the theory are not readily made.

---

[6] Charles Hitch, "Sub-optimization in Operations Problems," Journal of the Operations Research Society of America, Vol. 1 (1953), pp. 87-99.

Hitch's paper promises to become something of a classic in the operations research literature. The major point--that the "objectives" or 'payoff" functions devised for particular operations analyses must be correctly chosen if the analytical results are to be useful--rests on the fact that most O. R. problems involve a subordinate part of a larger operation. Thus, as is ordinarily the case in economic and business analyses, it is necessary to relate the particular operation under study to the overall objectives to which it contributes.

A related methodological point is less well recognized among businessmen, statisticians, and, unfortunately, even economists. No dynamic problem can be usefully formulated in terms of multiple superlatives. Thus, it is useful, no matter how "unrealistic," to formulate all business problems in terms of profit maximization for the firm as a whole, in order to emphasize the subordinate nature of all the interrelated processes involved. It will rarely pay, e.g., to maximize sales volume, or to minimize average stock levels per se. Such objectives would be mutually exclusive; and optimal sales volume and stock levels can only be determined through establishing their relationships to the profit function. Cf. J. von Neumann and O. Morgenstern, Theory of Games and Economic Behavior, Princeton, 1947, p. 11.

Whitin wisely does not attempt a formal solution to the style-goods case as defined above, but again concentrates on determination of optimal stock levels given a decision to stock a particular item at all, and given a price and estimated probability distribution of demand for the period. Ordering costs and carrying charges are unimportant, in this case, relative to profits lost by running out of a wanted item or the losses that must be taken on items unsold at the end of the period. By balancing these "expected costs" against one another Whitin determines the profit-maximizing number of items to stock. Compared with the case of highly standardized commodities suited to treatment under the general, simplified formulae, the cost elements involved are much more difficult to establish empirically, but again this appears to be a situation where the application of even an overly-simplified model promises substantial improvement over current practices.

NAVAL APPLICATION

Whitin's description and critical analysis of the Navy's current control system should be of interest to all service personnel, and to business managers as well. Chapter 8 reveals clearly and concisely the manner in which good theory competently applied to operating situations generates meaningful questions and helps to block out promising approaches to such questions.

A particular problem which receives attention here (pp. 182-184) is again a subordinate variant of the more general value-bound problem of the optimal composition of inventories. The theory of optimal inventory levels advanced by Whitin also allows some analysis of the cost consequences of diversifying an inventory or, conversely, of the savings from simplification and standardization. Optimal lot sizes and safety margins are directly related to the square root of volume demanded. A decision to stock four "models" or "styles" of a particular item, any one of which would be capable of serving total functional demand, will result in aggregate purchase quantities and inventory levels which are double the figure needed if a single item could be agreed upon as satisfactory. Viewed in reverse, this same relationship can be used to measure the cost, and thus better assess the net gain, to be had from the development of more highly specialized variants of a particular standardized item.

This last type of calculation is always a difficult one to make. It is perhaps worth pointing out that diversification is an important and logical strategy in any economy operating at high levels of specialization and division of labor. As products and services tend to become increasingly specialized to particular end-uses, they also tend to become specialized in a number of related, indivisible dimensions. Expected demand for any one such product is a matter of compound probabilities, while the demand for a family of such products is built up by summing probabilities and will therefore be less subject to skew and dispersion. This problem is discussed by Whitin, but largely in passing and on the basis of unrationalized empirical observation. A similar uncertainty-aversion strategy that can be important is revealed by examination of the multiple-stage or "cascading" problem. Given a family of "models" of a particular type of equipment, it is possible to reduce inventory levels and also the probability of obsolescence by accepting a longer lead time and storing interchangeable sub-assemblies. Required levels in this case will again benefit from the expected square-root relationship. This notion is closely related to the economists' and financial analysts' concept of "liquidity."

CONCLUSIONS

The Theory of Inventory Management, in part because it does not consider a number of the finer points that have been investigated by other students of inventory problems, is currently

the most useful single reference source available to those interested in the field. Whitin has been singularly successful in gauging, and writing to reduce, the large gap that still exists between inventory theory and practice. Though much remains to be done at all levels, this book will undoubtedly be granted a large measure of credit for having demonstrated the practical and immediate gains to be had from (a) careful blending of theoretical analysis and experienced observation and (b) cooperative pooling of governmental, university, and business resources in an attack on common problems.

* * *

MINIMIZATION OF NON-LINEAR SEPARABLE CONVEX FUNCTIONALS[1]

A. Charnes
Carnegie Institute of Technology
and
C. E. Lemke
General Electric Company

Programming problems may arise in which the variables are subject to linear equations and inequalities, but the objective function may not be a linear function of the variable. This paper shows how the methods of linear programming may be extended to cover any objective function which is the sum of convex functions of each of the variables. The technique consists of constructing a large linear program whose solution yields the solution to a polygonal approximation of the convex programming problem. An efficient computational algorithm for solving the auxiliary linear program is also presented.

1. INTRODUCTION

The minimization of non-linear convex functionals (or equivalently, maximization of concave functionals) subject to linear inequalities has generally been regarded as inaccessible to the methods of linear programming because the optimizing points are usually interior points rather than extreme points. In this paper we shall show that for a certain important class of these functionals, "separable convex functionals", a new extended "simplex method," in the announced spirit of this series [1], can be employed to yield a solution to an arbitrary degree of approximation.

By separable convex functional we mean a functional which, by a suitable linear transformation of variables, can be exhibited in the form:

$$f(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} f_j(x_j),$$

where $f_j(x_j)$ is a convex function of the single variable x_j.

Problems involving such functionals occur in numerous applications, e.g., the production-inventory model of Modigliani and Hohn, chemical processing models, oil-refining problems ("concave programming" considered by A. S. Manne, without developing a computational method), and economic theories elaborated by Koopmans, et al., in which one is to minimize a positive semi-definite quadratic form.

As with the "bounded variables" problem, we shall develop our method in the context of a single important example, least squares regression subject to linear inequality restraints, taking care, however, that the procedure is clearly outlined for even its most general instance. Again we achieve a notable reduction in the computational tableau through a new extension of the simplex technique to take advantage of the structure of the "uncompressed" matrix of the problem. In contrast to the "bounded variables" problem, however, where the direct problem is more transparent, here we rely heavily on the dual formulation.

---

[1]This paper was written as part of the project, "The Planning and Control of Industrial Operations" under Contract Nonr-760(01) with the Office of Naval Research.

2. THE PROBLEM OF CONSTRAINED LEAST SQUARES APPROXIMATION

Suppose that n observations on m+1 variates have been made. Let the mxn observations made on the first m (the independent) variates by a_{ij}, and those made on the (m+1)st variate be c_j. On the basis of these observations we seek a "best" linear relation for the a's and c's of the form:

$$\sum_{i=1}^{m} a_i w_i = c, \tag{1}$$

where, in addition, the w_i are constrained to satisfy the p inequalities:

$$\sum_{i=1}^{m} b_{ij} w_i \geq d_j \; ; \; j = 1, \ldots, p. \tag{2}$$

If we define the quantities:

$$\epsilon_j = \sum_{i=1}^{m} a_{ij} w_i - c_j \; ; \; j = 1, \ldots, n, \tag{3}$$

we agree to seek a set of w_i, subject to the restrictions (2), which minimizes the quadratic form:

$$Q(\epsilon) = \sum_{j=1}^{n} \epsilon_j^2. \tag{4}$$

If we replace the equalities (3) by the equivalent set of inequalities:

$$\begin{aligned} \sum_{i=1}^{m} a_{ij} w_i - \epsilon_j &\geq c_j \\ -\sum_{i=1}^{m} a_{ij} w_i + \epsilon_j &\geq -c_j \end{aligned} \quad ; \qquad j = 1, \ldots, n, \tag{5}$$

we see that this problem is of the form mentioned in the introduction; each of the functions $f_i(\epsilon_i) = \epsilon_i^2$ is a parabola opening upward in the (ϵ_i, f_i)- plane.

3. THE "SEPARABLE CONVEX FUNCTIONAL" PROBLEM

We proceed to a discussion of the solution of the problem mentioned in the introduction. Let us first introduce the m-space vectors P_j; $j = 1, \ldots, n$, where P_j is the mxl matrix whose ith entry is a_{ij}. Let x be the vector whose ith component is x_i. We may then phrase the problem as:

Minimize

$$F(x) = \sum_{i=1}^{m} f_i(x_i),$$

where

$$P_j'x \geq c_j ; \qquad j = 1, \ldots, n.$$

We shall do the following:

A. Replace the above problem by a linear one whose solution approximates the solution to the "separable" one.

B. By utilizing the dual theorem of linear programming, transform the approximating problem into a simplex problem.

C. Show how the bulk of the computations involved in solving the linear problem may be referred to the original matrix (a_{ij}), both computationally and theoretically.

We intend, in the future, to extend this method to applying the final solution to this approximating problem to the solving of successively better linear approximations.

A. The Approximating Problem

The linear approximating problem is set up as follows:

Each of the separable convex functions f_i is replaced by the piecewise linear function $g_i(x_i)$ shown in Figure 1.

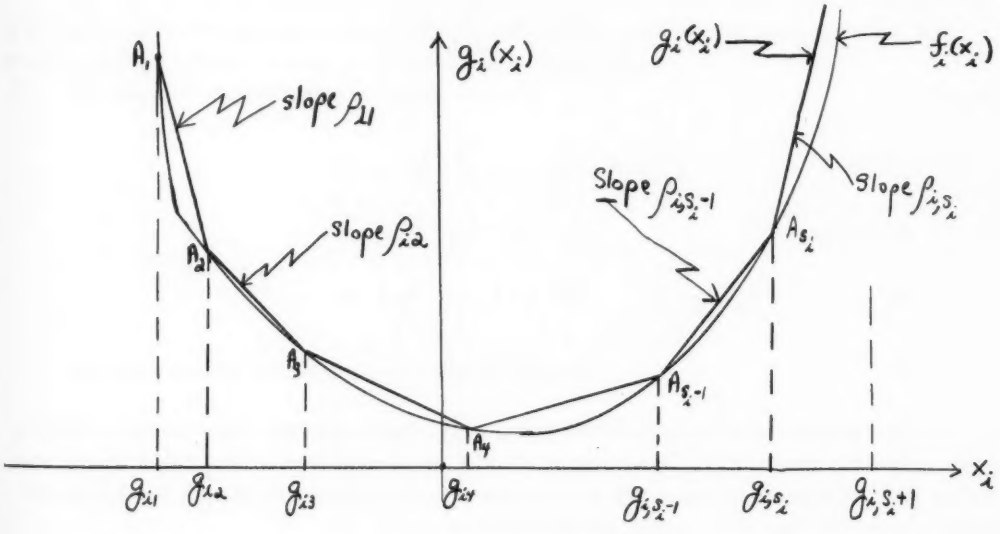


Figure 1

If we define the variables u_{ij} and v_{ij}, for $j = 1, \ldots, s_i$, by the conditions:

i. $$u_{ij} \geq 0 , \; v_{ij} \geq 0,$$

ii. $$x_i + u_{ij} - v_{im} = g_{ij},$$

iii. $$u_{ij}v_{ij} = 0,$$

each $g_i(x_i)$ may then be expressed analytically as:

$$g_i = \delta_i + \rho_i(x_i - v_{i1}) + \rho_{i1}(v_{i1} - v_{i2}) + \ldots + \rho_{i,s_i-1}(v_{i,s_i-1} - v_{i,s_i}) + \rho_{i,s_i} v_{i,s_i}, \tag{6}$$

where $\delta_i = f_i(g_{i1}) - \rho_i g_{i1}$, ρ_i is the slope of g_i to the left of A_1, and ρ_{ir} is the slope of the segment $A_r A_{r+1}$.

We may assume that g_{i1} has been chosen so far to the left that a value of x_i such that $x_i < g_{i1}$ will not enter into our computations. The constants δ_i may then be omitted, to be added in at the completion of the problem. We shall henceforth use $g_i(x_i)$ as in (6) but with δ_i omitted.

If we define $k_{i1} = \rho_{i1} - \rho_i$ and $k_{ij} = \rho_{ij} - \rho_{i,j-1}$, we can write:

$$g_i = \rho_i x_i + \sum_{j=1}^{s_i} k_{ij} v_{ij}. \tag{7}$$

Thus, an approximate solution to the original problem is found by solving the problem:
Minimize

$$G(x, v) = \sum_{i=1}^{m} g_i(x_i)$$

where

<u>i.</u> $\quad P_j' x \geq c_j ; \qquad j = 1, \ldots, n,$

<u>ii.</u> $\quad x_i = u_{ij} - v_{ij} = g_{ij},$

<u>iii.</u> $\quad u_{ij}, v_{ij} \geq 0 ; \qquad \text{for } j = 1, \ldots, s_i, \; i = 1, \ldots, m.$

<u>iv.</u> $\quad u_{ij} v_{ij} = 0.$

This is still not a linear problem owing to <u>iv</u>. Note, however, that the convexity of g_i implies that the quantities k_{ij} appearing in (7) will be nonnegative. If $k_{ij} > 0$, then any solution of <u>i</u>, <u>ii</u>, <u>iii</u> which minimizes $G(x,v)$ which will automatically satisfy <u>iv</u>. If $k_{ij} = 0$, then we are indifferent to the size of v_{ij}, hence indifferent to <u>iv</u>.

Hence we can write the equivalent linear problem:
Minimize

$$G(x, v) = \sum_{i=1}^{m} \rho_i x_i + \sum_{i=1}^{m} \sum_{j=1}^{s_i} k_{ij} v_{ij},$$

where

$$P_j'x \geq c_j,\ j = 1, \ldots, n,$$

$$-x_i + v_{ij} \geq -g_{ij}, \qquad j = 1, \ldots, s_i\ ;\ i = 1, \ldots, m. \tag{8}$$

$$v_{ij} \geq 0.$$

(8) then represents N linear inequalities in M unknowns where

$$N = n + 2\sum_{i=1}^{m} s_i$$

and

$$M = m + \sum_{i=1}^{m} s_i.$$

We next put (8) in terms of M-space vectors. A vector in M-space will be so denoted by placing a bar above the letter designation. At times we will want to write a vector $\bar{x}$ in M-space as $\bar{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, where x is in m-space and y is in (M-m)-space.

We shall use the following m-space vectors:

$$P_o = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_m \end{pmatrix} \quad ; \quad e_i = \begin{pmatrix} 0 \\ \cdot \\ 0 \\ 1_i \\ 0 \\ \cdot \\ 0 \end{pmatrix} \quad \begin{matrix} (1 \text{ in the ith place,} \\ 0 \text{ elsewhere.}) \end{matrix}$$

We shall use the following (M-m)-space vectors:

$$d = \begin{pmatrix} k_{11} \\ \cdot \\ \cdot \\ k_{1s_1} \\ k_{21} \\ \cdot \\ \cdot \\ k_{2s_2} \\ \cdot \\ \cdot \\ k_{m1} \\ \cdot \\ \cdot \\ k_{ms_m} \end{pmatrix} \quad ; \quad v = \begin{pmatrix} v_{11} \\ \cdot \\ \cdot \\ v_{1s_1} \\ v_{21} \\ \cdot \\ \cdot \\ v_{2s_2} \\ \cdot \\ \cdot \\ v_{m1} \\ \cdot \\ \cdot \\ v_{ms_m} \end{pmatrix},$$

and vectors y_{ij}, having a 1 in the $(j + \sum_{t=1}^{i-1} s_t)$th place and 0 elsewhere, for $j = 1, \ldots, s_i$; $i = 1, \ldots, m$.

In terms of these we want the M-space vectors:

$$\bar{e}_i = \begin{pmatrix} e_i \\ 0 \end{pmatrix}; \; i = 1, \ldots, m; \quad \bar{P}_j = \begin{pmatrix} P_j \\ 0 \end{pmatrix}; \; j = 1, \ldots, n; \quad \bar{P}_o = \begin{pmatrix} P_o \\ d \end{pmatrix},$$

$$\bar{U}_{ij} = \begin{pmatrix} 0 \\ y_{ij} \end{pmatrix}; \quad \bar{V}_{ij} = \bar{U}_{ij} - \bar{e}_i = \begin{pmatrix} -e_i \\ y_{ij} \end{pmatrix}; \; j = 1, \ldots, s_i; \; i = 1, \ldots, m,$$

$$\text{and } \bar{w} = \begin{pmatrix} x \\ v \end{pmatrix}.$$

In terms of these vectors (8) takes the form:

Minimize $\bar{P}_o' \bar{w}$ where:

$$\begin{aligned} &\bar{P}_j' \bar{w} \geq c_j \; , \; j = 1, \ldots, n, \\ &\bar{V}_{ij}' \bar{w} \geq -g_{ij} \quad , \; j = 1, \ldots, s_i \; ; \; i = 1, \ldots, m. \\ &\bar{U}_{ij}' \bar{w} \geq 0 \end{aligned} \tag{9}$$

B. Transformation to the Dual Problem

We now utilize the dual theorem of Tucker, Kuhn and Gale [2] which may be stated as follows:

Let $P_o, P_1, \ldots, P_n$ be given vectors and $c_1, c_2, \ldots, c_n$ be given real numbers. Consider the two dual problems:

I. Minimize $P_o' w$ where $P_j' w \geq c_j$; $j = 1, \ldots, n$.

II. Maximize $Z_o = \sum_{j=1}^{n} \rho_j c_j$ where $P_o = \sum_{j=1}^{n} \rho_j P_j$ and $\rho_j \geq 0$.

If either Min $P_o' w$ or Max Z_o is finite then Min $P_o' w$ = Max Z_o.

Hence, by the dual theorem, solving the dual problem to (9) will yield the common optimum. Since by our solution procedures we automatically obtain an extreme point solution to both this problem and the direct problem, there is no question of loss of equivalence. The dual problem to (9) is the simplex problem:

Maximize

$$Z_o = \sum_{j=1}^{n} \mu_j c_j + \sum_{i=1}^{m} \sum_{j=1}^{s_i} \gamma_{ij}(-g_{ij}),$$

where

$$\bar{P}_o = \sum_{j=1}^{n} \mu_j \bar{P}_j + \sum_{i=1}^{m} \sum_{j=1}^{s_i} \gamma_{ij} \bar{V}_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{s_i} \lambda_{ij} \bar{U}_{ij}, \tag{10}$$

and

$$\mu_j,\ \gamma_{ij},\ \lambda_{ij} \geq 0.$$

We shall solve this problem, which shall be called the M-space problem, by the simplex method discussed in [1], but we intend to show how the problem may be handled in m-space, using only the initial m-space vectors P_j, $j = 1, \ldots, n$ and the unit vectors e_i, $i = 1, \ldots, m$.

C. Solving the M-space problem in m-space.

Starting with an arbitrary basic solution to the M-space problem we discuss the transition to a new basic solution. The main steps in such a transition (see [1]) are:

C1. Deciding upon the replacing vector.

C2. Deciding upon the vector to be replaced.

C3. Computing sufficient data so that C1 and C2, for the new basic solution, may be accomplished.

We shall discuss these steps in that order, but first we may deduce some properties of an arbitrary basis B_M of vectors chosen from among the vectors $\bar{P}_j$, $\bar{U}_{ij}$ and $\bar{V}_{ij}$.

i. For fixed i and for each $j = 1, \ldots, s_i$ at least one of the vectors $\bar{U}_{ij}$ and $\bar{V}_{ij}$ are in B_M.

ii. Since i accounts for M-m vectors of B_M, at most m of the vectors $\bar{P}_j$ are in B_M. We may suppose these vectors are $\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_t$, $0 \leq t \leq m$.

iii. For precisely m-t values of i, say $i_1, i_2, \ldots, i_{m-t}$, there are values $j_1, j_2, \ldots, j_{m-t}$ such that both $\bar{U}_{i_k j_k}$ and $\bar{V}_{i_k j_k}$, $k = 1, \ldots, m-t$ are in B_M.

iv. The vectors $P_1, \ldots, P_t, e_{i_1}, e_{i_2}, \ldots, e_{i_{m-t}}$ form a basis B_M of m-space.

Let a^i, $i = 1, \ldots, m$ be the vectors dual to the vectors of B_M. Thus, the a^i are defined by the relations:

$$\begin{aligned} &P_i' a^j = \delta_{ij}\ ;\ i = 1, \ldots, t; \quad j = 1, \ldots, m, \\ &e_{i_k}' a^j = \delta_{kj}\ ;\ k = t+1, \ldots, m;\ j = 1, \ldots, m. \end{aligned} \tag{11}$$

Let w_o be the point satisfying:

$$\begin{aligned} &P_i' w_o = c_i\ ; \quad i = 1, \ldots, t, \\ &e_{i_k}' w_o = g_{i_k j_k}\ ;\ k = 1, \ldots, m-t. \end{aligned} \tag{12}$$

Now in particular if B_M is the basis corresponding to any basic solution of the M-space problem, we may show that having computed the vectors a^i and w_o we may proceed to a new basic solution, if necessary.

We first show how to express the vectors $\bar{P}_j$, $\bar{U}_{ij}$ and $\bar{V}_{ij}$ in terms of the basis B_M. We have:

$$\bar{P}_j = \sum_{i+1}^{t} (P_j' a^i)\bar{P}_i + \sum_{k=1}^{m-t} (P_j' a^{t+k})\bar{e}_{i_k} = \sum_{i=1}^{t} (P_j' a^i)\bar{P}_i + \sum_{k=1}^{m-t} P_j' a^{t+k}[\bar{U}_{i_k j_k} - \bar{V}_{i_k j_k}]. \tag{13}$$

$$\bar{U}_{ij} = \bar{V}_{ij} + \bar{e}_i = \bar{V}_{ij} + \sum_{p=1}^{t} (e_i' a^p)\bar{P}_p + \sum_{k=1}^{m-t} (e_i' a^{t+k})[\bar{U}_{i_k j_k} - \bar{V}_{i_k j_k}], \tag{14}$$

which holds for those $\bar{U}_{ij}$ not in B_M.

$$\bar{V}_{ij} = \bar{U}_{ij} - \bar{e}_i = \bar{U}_{ij} - \sum_{p=1}^{t} (e_i' a^p)\bar{P}_p - \sum_{k=1}^{m-t} (e_i' a^{t+k})[\bar{U}_{i_k j_k} - \bar{V}_{i_k j_k}], \tag{15}$$

which holds for those $\bar{V}_{ij}$ not in B_M.

C1. Deciding upon the replacing vector. For this we need the quantities $Z(\bar{P}_j)$, $Z(\bar{U}_{ij})$ and $Z(\bar{V}_{ij})$. Recall, as in [1], that these are obtained from the vector equations (13), (14) and (15), respectively, by replacing each basis vector on the right by its corresponding scalar. Thus we have:

$$Z(\bar{P}_j) = \sum_{i=1}^{t} (P_j' a^i)c_i + \sum_{k=1}^{m-t} (P_j' a^{t+k}) g_{i_k j_k} - c_j \tag{16}$$

$$= \sum_{i=1}^{t} (P_j' a^i)(w_o' a_i) + \sum_{k=1}^{m-t} (P_j' a^{t+k})(e_{i_k}' w_o) - c_j = P_j' w_o - c_j.$$

$$Z(\bar{U}_{ij}) = -g_{ij} + \sum_{r=1}^{t} (e_i' a^r)c_r + \sum_{k=1}^{m-t} (e_i' a^{t+k}) g_{i_k j_k} = e_i' w_o - g_{ij}. \tag{17}$$

$$Z(\bar{V}_{ij}) = -\sum_{r=1}^{t} (e_i' a^r)c_r - \sum_{k=1}^{m-t} (e_i' a^{t+k}) g_{i_k j_k} + g_{ij} = -(e_i' w_o - g_{ij}). \tag{18}$$

These are given only in terms of the P_j, w_o and the initial g_{ij}.

The quantities (16), (17) and (18) allow us to decide upon a replacing vector. If all of these quantities are nonnegative the problem is finished. If not we may take as replacing vector any $\bar{Q}$ with $Z(\bar{Q}) < 0$.

C2. Deciding upon the vector to be replaced. $\bar{Q}$ obtained above is to replace some vector in B_M to form a new basic solution. To decide upon the vector to be replaced, let $\bar{a}^i$, for $i = 1, \ldots, M$ be the vectors dual to the basis B_M. Then the components of $\bar{P}_o$ in terms of the basis vectors of B_M will be simply $(\bar{P}_o' \bar{a}^i)$ and those of $\bar{Q}$ will be $\bar{Q}' \bar{a}^i$.

We require the quantity θ defined by:

$$\theta = \operatorname{Min}_i \frac{\bar{P}_o' \bar{a}^i}{\bar{Q}' \bar{a}^i}, \tag{19}$$

for those values of i such that $\bar{Q}' \bar{a}^i > 0$.

Now there will either be a unique value of i yielding θ (corresponding to a non-degenerate stage) in which case $\bar{Q}$ will replace the ith vector of B_M, or else the value of θ is obtained for more than one value of i (corresponding to a degenerate stage) in which case the simple criterion devised by Charnes [4] is applicable, again yielding a unique value of i, so that $\bar{Q}$ replaces the ith vector of B_M determined by the quantity θ.

Now since we do not compute the vectors $\bar{a}^i$ directly, we must find the quantities $\bar{P}_o' \bar{a}^i$ by other means. They are given as the coefficients of the basis vectors in the expression for $\bar{P}_o$ in terms of the basis B_M. Thus:

$$\bar{P}_o = \begin{pmatrix} P_o \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ d \end{pmatrix} = \sum_{i=1}^{m} (P_o' a^i)\bar{P}_i + \sum_{k=1}^{m-t} P_o' a^{t+k} \left[\bar{U}_{i_k j_k} - \bar{V}_{i_k j_k} \right] + \sum_{i=1}^{m} \sum_{j=1}^{s_i} k_{ij} \bar{U}_{ij}. \tag{20}$$

From this it is clear that one obtains $\bar{P}_o$ in terms of B_M as soon as the vectors of that basis are known. Thus, if some $\bar{U}_{ij}$ is not in B_M it is replaced by its known expression given by (14).

It appears, however, that the most efficient way of obtaining the coefficients $\bar{P}_o' \bar{a}^i$ is by algorithm, as in the usual simplex procedure, from the previous tableau.

C3. Completing the transition to the new basic solution. Having decided upon the replacing vector and the vector to be replaced, the transition to the new basic solution to the M-space problem will be complete when the new corresponding m-space basis B_m' is determined and its dual set (the set of new a^i) and the new w_o are found.

For this, we need to discuss the various possibilities, depending upon whether $\bar{Q}$ is a $\bar{P}_j$, a $\bar{U}_{ij}$ or a $\bar{V}_{ij}$.

Case I: $\bar{Q} = \bar{P}_j$, with $t < j \leq n$, and $\bar{Q}$ replaces:

a. $\bar{P}_i$, with $1 \leq i \leq t$. Then P_j replaces P_i in the basis B_m.

b. $\bar{U}_{i_r j_r}$ or $\bar{V}_{i_r j_r}$ with $1 \leq r \leq m-t$. Then P_j replaces e_{i_r} in the basis B_m.

Case II: $\bar{Q} = \bar{U}_{ij}$ and replaces:

a. $\bar{V}_{ij}$. Then B_m does not change.

b. $\bar{P}_r$ with $1 \leq r \leq t$. Then e_i with corresponding scalar g_{ij} replaces P_r with corresponding scalar c_r.

c. $\bar{U}_{i_r j_r}$ or $\bar{V}_{i_r j_r}$ with $1 \leq r \leq m-t$. Then e_i with scalar g_{ij} replaces e_{i_r} with scalar $g_{i_r j_r}$.

Case III: $\bar{Q} = \bar{V}_{ij}$, and replaces:

a. $\bar{U}_{ij}$. Then the m-space basis does not change.

b. $\bar{P}_r$ with $1 \leq r \leq t$. Then e_i and g_{ij} replace P_r and c_r respectively.

c. $\bar{U}_{i_r j_r}$ or $\bar{V}_{i_r j_r}$ with $1 \leq r \leq m-t$. Then e_i and g_{ij} replace e_{i_r} and $g_{i_r j_r}$ respectively.

We have thus shown that essentially all of the computations in one stage of the M-space problem may be handled in terms of m-space vectors. As in [1], in the case of the bounded variables problem, to complete the discussion of the approximation problem we shall exhibit a computational tableau associated with one stage of the problem, and list the rules of procedure to be followed in passing to the next stage.

Extended Tableau No. 2

Tableau 2A

f	B_m	e_1	e_2	$\cdots$	e_m	
c_1	P_1	$e_1'a^1$	$e_2'a^1$	$\cdots$	$e_m'a^1$	0
.	.	.	.		.	.
.	.	.	.		.	.
c_t	P_t	$e_1'a^t$	$e_2'a^t$	$\cdots$	$e_m'a^t$	0
$g_{i_1 j_1}$	e_{i_1}	$e_1'a^{t+1}$	$e_2'a^{t+1}$	$\cdots$	$e_m'a^{t+1}$	0
.	.	.	.		.	.
.	.	.	.		.	.
$g_{i_{m-t} j_{m-t}}$	$e_{i_{m-t}}$	$e_1'a^m$	$e_2'a^m$	$\cdots$	$e_m'a^m$	0
	w_o	$e_1'w_o$	$e_2'w_o$	$\cdots$	$e_m'w_o$	1

Tableau 2B

0	+, μ_1	+, μ_2	$\cdots$	+, μ_t	-, $Z(\bar{P}_{t+1})$	$\cdots$	-, $Z(\bar{P}_n)$
1	1, 1	1, 2	$\cdots$	1, s_1			
2	2, 1	2, 2	$\cdots\cdots\cdots$		2, s_2		
.	.	.					
.	.	.					
.	.	.					
m	m, 1	m, 2	$\cdots\cdots$		m, s_m		

Explanations. Tableau 2A shows all of the computed data relevant to the stage in question—namely, the computed vectors a^i, whose components $e_j'a^i$ are given in the ith row of the tableau, and the vector w_o whose components are $e_j'w_o$.

Tableau 2B consists of m+1 rows. Row 0 contains n entries. The ith entry has a "+" sign and the quantity μ_i if P_i is in B_m (in the case under discussion, i = 1, . . . , t), or

the ith entry has a "-" sign and the quantity $Z(\bar{P}_i) = P_i'w_o - c_i$ if P_i is not in B_m (c. f. equation (10) for the μ_i).

The ith row, for each of the other m rows, contains s_i entries. The jth entry of the ith row is subdivided as follows:

+	$Z(\bar{V}_{ij})$	λ_{ij}	0

(cf. (10) for λ_{ij})

if $\bar{U}_{ij}$ is in B_M and $\bar{V}_{ij}$ is not, or:

-	$Z(\bar{U}_{ij})$	0	γ_{ij}

(cf. (10) for γi_j)

if $\bar{V}_{ij}$ is in B_M and $\bar{U}_{ij}$ is not, or:

+ -	0	λ_{ij}	γ_{ij}

if both $\bar{V}_{ij}$ and $\bar{U}_{ij}$ are in B_M.

Thus, each (i, j) entry has four subentries. We shall refer to the kth subentry, k = 1, 2, 3, 4, of the (i, j) entry as the (i, j, k) entry.

Rules of Procedure:

a. Having computed the a^i, i = 1, . . . , m, whose components are given in the ith row of Tableau 2A, we compute the vector w_o, whose components, $e_i'w_o$ are obtained as follows: From

$$w_o = \sum_{i=1}^{t} (P_i'w_o)a^i + \sum_{i=1}^{m-t} (e_{i_r}'w_o)a^{t+i}$$

we have

$$e_j'w_o = \sum_{i=1}^{t} (P_i'w_o)(e_j'a^i) + \sum_{i=1}^{m-t} (e_{i_r}'w_o)(e_j'a^{t+i})$$

$$= \sum_{i=1}^{m} c_i(e_j'a^i) + \sum_{i=1}^{m-t} g_{i_r j_r}(e_j'a^{t+i}).$$

Thus $e_j'w_o$ is obtained by taking the scalar product of f, whose components are in the first column of Tableau 2A, and the vector whose components are in the jth column. There is however a somewhat more efficient way of proceeding by the modified simplex method; a way which eliminates the necessity of computing w_o. This will appear in a later paper.

b. We next compute the quantities $Z(\bar{P}_j)$ for j = t+1, . . . , n according to formula (16). These are placed in the jth entry of the row 0, Tableau 2B. Likewise the quantities $Z(\bar{U}_{ij})$ and $Z(\bar{V}_{ij})$, computed via (17) and (18) respectively, are placed in entries (i, j, 2) of Tableau 2B.

c. We next need the quantities μ_j, λ_{ij} and γ_{ij}. These are to be obtained from the corresponding components $\bar{\lambda}_{ij}$ and $\bar{\gamma}_{ij}$ of the preceding tableau by the usual simplex method

algorithm. Having computed these quantities, and knowing by b. the replacing vector $\bar{Q}$, we can compute the quantity θ, by formula (19). This value will determine the vector to be replaced.

d. Having determined the replacing vector and the vector to be replaced we need to compute the components of the vectors of the new m-space inverse according to the criteria of C3. These vectors are computed, by algorithm, from the vectors a^i of the current m-space inverse in the usual simplex method fashion. The vector algorithm is given at the end of the first of the series of papers of [1].

Note, finally, that the entries (i, j, 1), Tableau 2B tell precisely which vectors are in the M-space basis B_M; "+" if $\bar{U}_{ij}$ is in B_M, "-" if $\bar{V}_{ij}$ is and "±" if both are.

REFERENCES

[1] Charnes, A., and Lemke, C. E., "Computational Theory of Linear Programming, I: The Bounded-variables Problem," O. N. R. Research Memorandum No. 10, Graduate School of Industrial Admin., Carnegie Inst. of Tech., January 1954.

[2] Gale, D., Kuhn, H. W., and Tucker, A. W., Activity Analysis of Production and Allocation, Chapter 19, Cowles Commission Monograph 13, John Wiley and Sons, New York, 1951.

[3] Charnes, A., Cooper, W. W., and Henderson, A., Introduction to Linear Programming, John Wiley and Sons, 1953.

[4] Charnes, A., "Optimality and Degeneracy in Linear Programming," Econometrica. Vol. 20 (1952), pp. 169.

* * *

FICTITIOUS PLAY FOR CONTINUOUS GAMES

J. M. Danskin[1]
Massachusetts Institute of Technology

This paper shows that a certain iterative process, due to G. Brown and Julia Robinson for finding the value of a finite game, converges also for continuous games.

1. INTRODUCTION

G. W. Brown conjectured, and Julia Robinson proved in [1], the validity of an iterative process for finding the value of a finite zero-sum two-person game. It is the purpose of this paper to show that the corresponding process is valid for zero-sum two-person games with continuous payoffs, played over the direct product of arbitrary compact spaces.

Let $M(x,y)$ be a continuous function defined on the direct product of two compact spaces $\mathfrak{X}$ and $\mathfrak{Y}$. We define sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ as follows:

$$\langle x_n \rangle\text{: } x_0 \text{ is arbitrary; } x_n \text{ maximizes the sum } \sum_{k=0}^{n-1} M(x, y_k) \ (n \geq 1);$$

$$\langle y_n \rangle\text{: } y_0 \text{ is arbitrary; } y_n \text{ minimizes the sum } \sum_{k=0}^{n-1} M(x_k, y) \quad (n \geq 1).$$

We shall prove that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} M(x_n, y_k) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} M(x_k, y_n) = v,$$

where v is the value of the game.[2] Our proof will be modelled on Mrs. Robinson's.

We shall give the proof for compact metric spaces. That this implies the result for compact spaces is seen as follows: if x and x' are elements of $\mathfrak{X}$ put

$$d(x, x') = \sup_{y \in \mathfrak{Y}} \left| M(x, y) - M(x', y) \right| .$$

On identifying any pair x, x' of elements of $\mathfrak{X}$ whose distance $d(x, x')$ is zero we get a metric space $\mathfrak{X}^*$, evidently compact. With an analogous definition of distance and identification we obtain from $\mathfrak{Y}$ a compact metric space $\mathfrak{Y}^*$. It is clear that if the result is true for all continuous games over $\mathfrak{X}^* \times \mathfrak{Y}^*$ then it is true for all continuous games over $\mathfrak{X} \times \mathfrak{Y}$.

The proof is here actually written out only for the case in which $\mathfrak{X}$ and $\mathfrak{Y}$ are both the unit interval, so that the game is played over the square. However, as no property of the unit

---

[1] This paper was written while the author was at the Rand Corporation. He is now with the Operations Evaluation Group of MIT.

[2] For the terminology of game theory, see for example [2].

interval except compactness plays a rôle, the argument goes through in the general case with only the obvious reinterpretations of the words row, column, and unit square.

Sections 2 to 4 will be devoted to a proof of the result announced. Section 5 contains some remarks on the result.

2. PRELIMINARIES—(r, s) GAMES

It follows from the minmax principle[3] that

$$\frac{1}{n}\sum_{k=0}^{n-1} M(x_n, y_k) \geq v \geq \frac{1}{n}\sum_{k=0}^{n-1} M(x_k, y_n);$$

accordingly it will clearly suffice to prove the following

LEMMA 4:[4] For any $\epsilon_0 > 0$ there is an n_0 such that

$$\frac{1}{n}\sum_{k=0}^{n-1} M(x_n, y_k) - \frac{1}{n}\sum_{k=0}^{n-1} M(x_k, y_n) < \epsilon_0$$

whenever $n \geq n_0$.

We choose such an ϵ_0; this will remain fixed throughout the sequel. Let now $0 < \epsilon_1 < \epsilon_0$, and divide the unit square into R^2 squares in such a way that the oscillation of M(x, y) on each square does not exceed $\frac{\epsilon 1}{2}$.

This subdivision induces a subdivision of the unit interval of the x-axis and the unit interval on the y-axis into R segments each. We shall refer to each of the R strips, parallel to the x-axis, which correspond to segments on the y-axis, as a row. Each of the R strips, parallel to the y-axis, corresponding to segments on the x-axis, will be called a column.

We shall use the locution "x is in the column" to mean "x is in the segment to which the column corresponds," and "the column containing x" to mean "the column corresponding to the segment containing x."

Consider now a game obtained from the original game by restricting x to the closure of r given columns and y to the closure of s given rows. Such a game will be called an (r, s) game. We introduce initial functions $S_0(x)$ and $I_0(y)$, gotten from continuous functions by restriction to the closures of the appropriate columns and rows, and satisfying the condition

$$\max S_0(x) = \min I_0(y) \tag{1}$$

on the restricted domains. We consider sequences $\langle x_n \rangle$ and $\langle y_n \rangle$, and functions $S_n(x)$ and $I_n(y)$, defined as follows:

---

[3] See for example [2]. The inequality used here is the trivial direction of the minmax theorem. See Section 5, Remark 5.1.

[4] We number Lemmas 1-4 in logical order rather than chronological order.

$$x_n \text{ maximizes } S_n(x), \text{ where } S_n(x) = S_0(x) + \sum_{k=0}^{n-1} M(x, y_k);$$

$$y_n \text{ minimizes } I_n(y), \text{ where } I_n(y) = I_0(y) + \sum_{k=0}^{n-1} M(x_k, y).$$

We are now prepared to state Lemma 3.

LEMMA 3: Let r and s be any integers satisfying $1 \leqq r \leqq R$ and $1 \leqq s \leqq R$. Then there exists an ϵ_2, satisfying $0 < \epsilon_2 < \epsilon_0$, and an n_0, such that for any (r, s) game and any choice of initial functions satisfying (1),

$$S_n(x_n) - I_n(y_n) < n\epsilon_2 \tag{2}$$

whenever $n \geqq n_0$.

It is clear that Lemma 3 implies Lemma 4; we have only to put $r = s = R$ and $S_0(x) = I_0(y) = 0$. Sections 3 and 4 will be devoted to the proof of Lemma 3. First, in Section 3, we prove two lemmas.

3. CONTINUATION—TWO LEMMAS

The results of this section, as stated in Lemma 2, are valid for any (r, s) game with any initial functions $S_0(x)$ and $I_0(y)$ satisfying (1). Observe that the quantity on the right hand side of (5) in Lemma 2 depends neither on r, nor s, nor the particular (r, s) game under consideration, nor on the choice of the initial functions.

This understood, we now fix attention on a particular (r, s) game. Lemmas 1 and 2 will be stated with respect to this game; in particular we ignore all rows and columns from which y and x, respectively, are excluded.

A row will be said to be eligible at the n^{th} stage provided its closure contains a y such that $I_n(y) = I_n(y_n)$. Similarly, a column will be said to be eligible at the n^{th} stage provided its closure contains an x such that $S_n(x) = S_n(x_n)$.

A row or column will be said to be eligible on the interval (n, n+p) (p an integer) provided it is eligible at the $n+\delta^{th}$ stage for some δ satisfying $0 \leqq \delta \leqq p$.

Let K be the least upper bound of $|M(x, y)|$ on the unit square.

LEMMA 1: If all the rows and columns are eligible on the interval (n, n+p), then

$$S_{n+p}(x_{n+p}) - \min \sum_{k=0}^{n+p-1} M(x, y_k) < 2pK + \frac{\epsilon_1}{2}(n+p) + S_0(x'')$$

and

$$\max \sum_{k=0}^{n+p-1} M(x_k, y) - I_{n+p}(y_{n+p}) < 2pK + \frac{\epsilon_1}{2}(n+p) - I_0(y''),$$

where ϵ_1 is the number defined in the second paragraph of Section 2, x'' is some number in an eligible column, and y'' is some number in an eligible row.

PROOF: We shall prove only the second of these inequalities. The proof of the first is analogous.

Let y' yield the maximum for the sum $\sum_{k=0}^{n+p-1} M(x_k, y)$. Now the row containing y' is eligible at the $n+\delta^{th}$ stage, where δ satisfies $0 \leqq \delta \leqq p$. Accordingly, for some y'' in that row, $I_{n+\delta}(y'') = I_{n+\delta}(y_{n+\delta})$. It follows that

$$\max \sum_{k=0}^{n+p-1} M(x_k, y) - I_{n+\delta}(y_{n+\delta})$$

(3)

$$= \sum_{k=0}^{n+p-1} M(x_k, y') - I_{n+\delta}(y'')$$

$$= \sum_{k=0}^{n+\delta-1} [M(x_k, y') - M(x_k, y'')] + \sum_{k=n+\delta}^{n+p-1} M(x_k, y') - I_0(y'')$$

$$\leqq \frac{\epsilon_1}{2}(n+\delta) + (p-\delta)K - I_0(y'')$$

$$\leqq \frac{\epsilon_1}{2}(n+p) + pK - I_0(y'') .$$

Now it is obvious that

$$I_{n+\delta}(y) - I_{n+p}(y) \leqq pK$$

for all y, and therefore in particular

$$I_{n+\delta}(y_{n+p}) - I_{n+p}(y_{n+p}) \leqq pK .$$

But from the choice of y_{n+p}, $I_{n+\delta}(y_{n+p}) \geqq I_{n+\delta}(y_{n+\delta})$. Hence

(4)

$$I_{n+\delta}(y_{n+\delta}) - I_{n+p}(y_{n+p}) \leqq pK .$$

The desired inequality results from the addition of (3) and (4).

LEMMA 2: If all the rows and columns are eligible on the interval (n, n+p), then

$$S_{n+p}(x_{n+p}) - I_{n+p}(y_{n+p}) \leq 4pK + \epsilon_1 (n+p) , \tag{5}$$

where ϵ_1 is as before, satisfying $0<\epsilon_1<\epsilon_0$.

PROOF: Consider the "transposed" game with payoff $M(x, y)$ and x and y restricted as before, but now with x the minimizing player and y the maximizing player. It follows from the minmax principle[5] that

$$\min \frac{1}{n+p} \sum_{k=0}^{n+p-1} M(x, y_k) \leq v' \leq \max \frac{1}{n+p} \sum_{k=0}^{n+p-1} M(x_k, y) ,$$

where v' is the value of the transposed game. Hence

$$\min \sum_{k=0}^{n+p-1} M(x, y_k) \leq \max \sum_{k=0}^{n+p-1} M(x_k, y) . \tag{6}$$

On the other hand, it follows from (1) that

$$S_0(x'') \leq I_0(y'') . \tag{7}$$

The desired inequality follows on adding the inequalities proved in Lemma 1 and taking account of (6) and (7).

We are now prepared to prove Lemma 3.

4. PROOF OF LEMMA 3

The statement of Lemma 3 will be found in Section 2. We shall make the proof by a step-by-step process.

PROOF: For $r=s=1$ the statement is obvious. Suppose that it holds for all $(r-1, s)$ and $(r, s-1)$ games. Let ϵ_3 be larger than ϵ_1 and larger than the ϵ_2's corresponding to the $(r-1, s)$ and $(r, s-1)$ games, but less than ϵ_0. Take n^* to be the larger of the n_0's corresponding to the $(r-1, s)$ and $(r, s-1)$ games. We shall show that the statement then holds for all (r, s) games, with ϵ_2 replaced by $\frac{\epsilon_0 + \epsilon_3}{2}$ and n_0 given by

$$n_0 = \left[\frac{8K}{\epsilon_0 - \epsilon_3} + 1 \right] \times n^* . \tag{8}$$

We turn now to a consideration of the effect of an ineligible row or column of an (r, s) game. Suppose for example that some row is ineligible on the interval $(m, m+n^*)$. Now consider the $(r, s-1)$ game gotten by suppressing that row, and with the following choices for initial functions:

---

[5] See Footnote 3.

$$S'_o(x) = S_m(x) + I_m(y_m) - S_m(x_m)$$

$$I'_o(y) = I_m(y) .$$

Observe that these functions, on their restricted domains, satisfy the condition (1). Define sequences $< x'_j >$ and $< y'_j >$ as follows:

(9) $$x'_j = x_{m+j} \text{ and } y'_j = y_{m+j} \quad (j = 0, 1, \ldots, n^*) .$$

Let us write, as usual,

$$S'_j(x) = S'_o(x) + \sum_{k=0}^{j-1} M(x, y'_k)$$

and

$$I'_j(y) = I'_o(y) + \sum_{k=0}^{j-1} M(x'_k, y) , \quad (j = 0, 1, \ldots, n^*) .$$

Evidently,

(10) $$S'_j(x) = S_{m+j}(x) + I_m(y_m) - S_m(x_m)$$

and

(11) $$I'_j(y) = I_{m+j}(y) .$$

Now since x_{m+j} maximizes $S_{m+j}(x)$, it follows that x'_j maximizes $S'_j(x)$ for $j = 0, 1, \ldots, n^*$. Recalling that for $0 \leq j \leq n^*$ no y_{m+j} entered the suppressed row, we see similarly that y'_j minimizes $I'_j(y)$ for $j = 0, 1, \ldots, n^*$. Accordingly the x'_j and y'_j given by (9) are the first n^*+1 terms of admissible (i.e., defined by (9)) infinite sequences $<x'_j>$ and $<y'_j>$ $(j = 0, 1, \ldots, n^*, n^*+1, \ldots)$ for the (r, s-1) game. Applying Lemma 3 to the (r, s-1) game, we thus obtain

(12) $$S'_{n^*}(x'_{n^*}) - I'_{n^*}(y'_{n^*}) < n^*\epsilon_3 .$$

On taking account of (10) and (11) there results

(13) $$S_{m+n^*}(x_{m+n^*}) - I_{m+n^*}(y_{m+n^*}) < S_m(x_m) - I_m(y_m) + n^*\epsilon_3 .$$

As the same argument, mutatis mutandis, works also in the case of an ineligible column, we have thus proved that if on an interval of length n^* there is an ineligible row or column, then the increase in the difference $S_n(x_n) - I_n(y_n)$ across the interval is less than $n^*\epsilon_3$.

Now let us take an n satisfying $n \geqq n_0$, n_0 defined as in (8), and write $n = n^*(q + \theta)$, where $0 \leqq \theta < 1$ and q is an integer. Denote by Z_ρ the interval $[n^*(\rho+\theta-1),\ n^*(\rho+\theta)]$ $(\rho = 1, 2, \ldots, q)$.

Two cases arise:

Case I: There is a ρ_0, and so a largest ρ_0, such that all the rows and columns are eligible on Z_{ρ_0}. Then, from Lemma 2 and (13), we get

$$S_n(x_n) - I_n(y_n) \leqq 4n^*K + \epsilon_1(\rho_0 + \theta)n^* + \epsilon_3(q - \rho_0)n^* \tag{14}$$

$$\leqq 4n^*K + n\epsilon_3$$

Case II: Each Z_ρ has an ineligible row or column. Observe first that

$$S_{n^*\theta}(x_{n^*\theta}) - I_{n^*\theta}(y_{n^*\theta}) \leqq 2n^*\theta K, \tag{15}$$

since $S_0(x_{n^*\theta}) - I_0(y_{n^*\theta}) \leqq 0$ and each of the $n^*\theta$ remaining terms appearing in each sum is bounded by K. Now, applying (13) to get across the q intervals Z_ρ $(\rho = 1, 2, \ldots, q)$ we obtain

$$S_n(x_n) - I_n(y_n) < 2n^*\theta K + q\epsilon_3 n^* \leqq 2n^*\theta K + \epsilon_3 n. \tag{16}$$

In either case, we have therefore

$$S_n(x_n) - I_n(y_n) \leqq 4n^*K + n\epsilon_3 \tag{17}$$

Now, since $q > \frac{n}{n^*} - 1$, then, from (8), on recalling that $n \geqq n_0$, we obtain

$$q > \frac{8K}{\epsilon_0 - \epsilon_3}. \tag{18}$$

It follows that

$$\frac{S_n(x_n) - I_n(y_n)}{n} \leqq \frac{4K}{q} + \epsilon_3$$

$$< \frac{\epsilon_0 - \epsilon_3}{2} + \epsilon_3 = \frac{\epsilon_0 + \epsilon_3}{2}$$

as asserted. This completes the proof of Lemma 3, and so the proof of the theorem announced in Section 1, for the case in which X and Y are both the unit interval. As we stated in Section 1, the proof of the general case requires only the obvious reinterpretation of the words row, column, and unit square.

5. REMARKS

5.1 In the proof of Lemma 4 we made no essential use of the minmax principle, only the trivial inequality

$$(19) \qquad \max_F \min_G \iint M(x, y)\, dF(x)\, dG(y) \leqq \min_G \max_F \iint M(x,y)\, dF(x)\, dG(y),$$

F and G varying among the spaces of distribution functions over X and Y respectively. But it is not difficult to see that Lemma 4 implies the inequality

$$(20) \qquad \min_G \max_F \iint M(x, y)\, dF(x)\, dG(y) \leqq \max_F \min_G \iint M(x, y)\, dF(x)\, dG(y),$$

and thus the minmax principle. Thus we have incidentally obtained another proof of that principle for the case of continuous games played over the direct product of arbitrary compact spaces. Lemma 4 and the minmax theorem then imply the result of Section 1.[6]

I am indebted to Oliver Gross of RAND for the two following remarks.

5.2 The present result implies that of Mrs. Robinson. Several proofs will occur to the reader. For example, a proof may easily be made employing the concept of (r, s) games.

5.3 There are simple examples to show that the result of Section 1 does not hold in the discontinuous case. One point of discontinuity will suffice for a game over the unit square.

REFERENCES

[1] Robinson, Julia, "An Iterative Method of Solving a Game," Annals of Mathematics, Vol. 54 (1951), pp. 296-301.

[2] Contributions to the Theory of Games, Annals of Mathematics Study No. 24, Princeton University Press, 1950.

* * *

[6] After this paper was written, J. G. Wendel has in fact (unpublished 1952) proved the following sharper result: let $F_n(x)$ and $G_n(y)$ be the distribution functions defined by the first n members of the x- and y- sequences, i.e., F_n associates the mass $1/n$ with each of the points $x_0, x_1, \ldots, x_{n-1}$, and similarly for G_n. Let F and G be weak star limit points of the sequences $\{F_n\}$ and $\{G_n\}$, the existence being guaranteed by weak star compactness. Then F and G are optimal strategies for the first and second players respectively.

AN ELEMENTARY TREATMENT OF AN INPUT-OUTPUT SYSTEM[1]

Y. K. Wong
Princeton University

This paper presents a mathematical study of an input-output system in which the outputs dominate the inputs. The method consists in using Cramer's rule for the solution of linear equations, Cauchy's expansion of a determinant and some elementary properties of inequalities. The fundamental results are stated in the Lemma in Section 1. Properties of the adjoint elements are established in Section 2. The operations are rational. The results hold for an ordered field.

INTRODUCTION

The mathematical treatments in the literature of input-output systems utilize Frobenius' results on characteristic values and vectors of non-negative matrices, and even Brouwer's fixed point theorem. In this note, we show that, by using some elementary methods, we obtain more results than by using those relatively advanced mathematical tools. Most of (if not all) the equations arising from linear economic systems are of an elementary nature and of a special type. The depth and generality underlying the abstract beauty of a mathematical system, such as the group theory, are lacking in the economic equations. It is, therefore, essential to bring out the special properties of a typical system of equations as formulated in economics. This end can be accomplished by means of elementary methods. The restriction to elementary methods is highly desirable in mathematics, although sometimes difficult to achieve. It is particularly important in applications.

1. AN ELEMENTARY MATHEMATICAL TREATMENT OF A LINEAR ECONOMIC SYSTEM

The equations arising from a linear economic system are of the form

$$x_i - \sum_{j=1}^{n} a_{ij}x_j = h_i \qquad (i = 1, 2, \ldots, n), \tag{1.1}$$

where

$$0 \leq a_{ij}, \qquad a_{ii} < 1, \qquad 0 \leq h_i . \tag{1.2}$$

In most cases, we have the column-sums

$$s_j = \sum_{i=1}^{n} a_{ij} \leq 1 . \tag{1.3}$$

---

[1]This paper was prepared under Office of Naval Research Contract N6onr-27009, and was part of a course on mathematics for economists given at Princeton University.

We may consider an ordered field, or simply the rational numbers. Thus, all quantities we use are abstract numbers without any physical or economical values attached to them. The output of each industry is represented by 1. The input for industry i purchased from industry j is given by a_{ij}, which is non-negative. Thus a_{ii} is the (direct) intra-industry transaction for industry i. (The indirect intra-industry transaction for industry i is expressible in terms of $n^2 - 1$ numbers, a_{pq}, where p, q = 1, 2, . . . , n with the number a_{ii} omitted.) The inequality $\sum_{i=1}^{n} a_{ij} \leq 1$ means that the sum of the inputs for industry j does not exceed its output.

Although the property that det (I-A) > 0 may hold without the validity of (1.3), we consider (1.3) as a very simple property and the column-sums are easy to compute. Moreover, (1.3) possesses an immediate economic interpretation, i.e., the outputs dominate the inputs. In what space do the observational data lie, and what is the metric we use? These two questions are important in the study of proper values of a matrix.[2] Hence, it is also important in the economic interpretation. In view of the fact that these difficulties arise in regard to proper values, we prefer to study the system of equations (1.1) with properties (1.2) and (1.3).

Since the equations (1.1) with properties (1.2) and (1.3) are formulated in such an elementary manner, it seems advisable to employ primarily elementary methods. Consider the question of the existence of a positive solution, namely, the restriction imposed on h_i or on a_{ij} in order that (1.1) has a positive solution. The following statement gives an answer to this question.

THEOREM. Suppose $h_i > 0$ for all i. The system of equations (1.1) with properties (1.2) and (1.3) has a unique positive solution (i.e., $x_i > 0$ for all i) if and only if $\det(I - A) \neq 0$.

A necessary and sufficient condition such as the one given in the above theorem is so ordinary that one may wonder why we even state it. It certainly lacks the "charm" of fixed point theorems, or even of characteristic values. Nevertheless, we are going to study the theorem, and shall use only elementary methods.

If an inhomogeneous system of linear equations

$$\sum_{j=1}^{n} c_{ij} u_j = w_i, \quad i = 1, \ldots, n$$

has a unique non-trivial solution, then it is necessary that $\det(c_{ij}) \neq 0$. This necessary condition can be found in many textbooks on college algebra. The sufficient condition can be treated in a very simple way by considering Cramer's rule. First of all, introduce some notations. As customary, let $\delta_{ij} = 1$ or 0 whenever $i = j$ or $i \neq j$. Let $b_{ij}^{(k)}$ be the adjoint elements[3] corresponding to the coefficients of (1.1) in which n is replaced by k. Let $d_k = \det(\delta_{ij} - a_{ij})$ for i, j = 1, 2, . . . , k.

---

[2] Cf. Y. K. Wong: "Some Mathematical Concepts for Linear Economic Models," (Sections 4-5, pp. 298-315) in Economic Activity Analysis, O. Morgenstern, ed., New York: John Wiley and Sons, Inc., 1954.

[3] See any text on college algebra.

LEMMA. If the coefficients a_{ij} $(i, j = 1, 2, \ldots, n)$ satisfy (1.2) and (1.3), then every adjoint element $b_{ij}^{(n)}$ is non-negative. Moreover, for $j \neq i$,

$$b_{ij}^{(n)} \leq b_{ii}^{(n)} \tag{1.4}$$

and

$$0 \leq d_k \leq (1 - a_{kk})d_{k-1} \qquad (k = 2, \ldots, n). \tag{1.5}$$

If $d_n \neq 0$, then all the principal minors d_k are greater than zero.

Proof of the theorem. We need only a weaker form of the lemma in order to prove the theorem. We need the non-negativeness of the adjoint elements, and the positiveness of diagonal elements, $b_{ii}^{(n)}$, of the adjoint.

As $d_n \neq 0$ and $d_n \geq 0$, we have $d_n > 0$.

Thus, by Cramer's rule,

$$x_i = \sum_{j=1}^{n} \frac{b_{ij}^{(n)}}{d_n} h_j > 0$$

as we wish to prove. The uniqueness is evident. This gives the sufficiency.

Proof of the lemma. The lemma is true for $n = 1, 2$. Assume the validity of the lemma for $k < n$. From (1.3), we have

$$a_{nj} \leq 1 - \sum_{i=1}^{n-1} a_{ij} = \sum_{i=1}^{n-1} (\delta_{ij} - a_{ij}) . \tag{1.6}$$

By the hypothesis of induction, $b_{jk}^{(n-1)}$ are nonnegative. Multiplying both sides of (1.6) by $b_{jk}^{(n-1)}$ and summing for $j = 1, 2, \ldots, n-1$, we have

$$\Sigma_j a_{nj} b_{jk}^{(n-1)} \leq \Sigma_i \Sigma_j (\delta_{ij} - a_{ij}) b_{jk}^{(n-1)} = d_{n-1} , \tag{1.7}$$

where all the summations extend from 1 to n-1. Now it is a simple matter to verify that

$$b_{nk}^{(n)} = \Sigma_j a_{nj} b_{jk}^{(n-1)} . \tag{1.8}$$

To prove the preceding equality, let c_{kn} be the minor obtained by striking out the kth row and nth column from $d_n = \det(a_{ij})$, $i, j = 1, 2, \ldots, n$. Then

$$b_{nk}^{(n)} = (-1)^{n+k} c_{kn} . \tag{1.9}$$

Let m_{kj} be the minor obtained by omitting the kth row and jth column from the first (n-1)th rows and columns of d_n. Then $b_{jk}^{(n-1)} = (-1)^{j+k}m_{kj}$. Now expand the minor c_{kn} according to the elements $-a_{n1}, -a_{n2}, \ldots, -a_{n,n-1}$ in the last row. Then

$$(1.10)\quad c_{kn} = \sum_{j=1}^{n-1} -a_{nj}(-1)^{n+j-1}m_{kj} = \sum_{j=1}^{n-1} -a_{nj}(-1)^{n+j-1}(-1)^{j+k}b_{jk}^{(n-1)} = \sum_{j=1}^{n-1} a_{nj}(-1)^{n+k}b_{jk}^{(n-1)} .$$

Combining (1.9) and (1.10), we obtain (1.8). Since $d_{n-1} = b_{nn}^{(n)}$, we have $0 \leq b_{nk}^{(n)} \leq b_{nn}^{(n)}$ by virtue of (1.8) and (1.7). Thus, we have proved that every adjoint element in the nth row is nonnegative. For general i, we can either change the notations slightly in the above proof, or interchange the ith, nth rows and also the ith, nth columns.

By Cauchy's expansion,

$$(1.11)\quad d_k = (1 - a_{kk})d_{k-1} - \sum_{r,s=1}^{k-1} a_{kr}b_{rs}^{(k-1)}a_{sk} .$$

Since all the adjoint elements $b_{rs}^{(k-1)}$ are nonnegative, we have

$$(1.12)\quad d_k \leq (1 - a_{kk})d_{k-1} .$$

This completes the proof of the lemma.

COROLLARY 1. If $\sum_{r=1}^{j} a_{rj} < 1$ for $j = 1, 2, \ldots, n$, then $d_n > 0$.

PROOF: Substituting (1.7) into (1.11) with $k = n$, we have

$$d_n \geq (1 - \sum_{r=1}^{n} a_{rn})d_{n-1} .$$

Now $d_2 \geq (1 - a_{22} - a_{12})d_1$ and $d_1 = 1 - a_{11} > 0$. By induction on n, we have the corollary. We can also prove the corollary by observing that

$$d_n \geq (1 - \sum_{r=1}^{n} a_{rn}) \ldots (1 - a_{22} - a_{12})(1 - a_{11}) .$$

COROLLARY 2. If $d_n > 0$, then $\sum_{r=1}^{n} a_{rj} < 1$ for $j = 1, 2, \ldots, n$, subject to the permutations of rows and the corresponding columns.

PROOF: Consider

$$(1.13)\quad x_j - \sum_{i=1}^{n} x_i a_{ij} = 0 \qquad (j = 1, 2, \ldots, n) .$$

If all the column-sums s_j defined by (1.3) are equal to 1, then $x_1 = x_2 = \ldots = x_n = 1$ would be a solution for the homogeneous equations (1.13). However, since, by hypothesis, $d_n > 0$, this would be impossible. Hence, there must be at least one column-sum, say s_n, which is less than unity. Now by (1.12), $d_n > 0$ implies that all principal minors d_k ($k < n$) are positive. Hence, applying the preceding argument to d_k ($k = n-1, n-2, \ldots, 2, 1$), we have the corollary.

We have, thus, established a very simple criterion for the determinant to be non-vanishing. If the determinant is non-vanishing, then it is positive by virtue of (1.5).

2. PROPERTIES OF THE ADJOINT ELEMENTS

In this section, we shall determine some relations between the elements in the same row of the adjoint $B = (b_{ij}^{(n)})$.

(I) *If all* $s_k < 1$ *for* $k = 1, 2, \ldots, n$, *then for* $j \neq i$, $b_{ij}^{(n)} < b_{ii}^{(n)}$, *where* $i, j = 1, 2, \ldots, n$.

Proof. The condition in Corollary 1 is satisfied, and hence $d_n > 0$, which implies that $b_{kk} > 0$ for all k. For the balance of our proof, let $i = n$. Since

$$a_{nk}b_{kk}^{(n-1)} < (1 - \sum_{r=1}^{n-1} a_{rk})b_{kk}^{(n-1)} \tag{2.1}$$

and

$$a_{nj}b_{jk}^{(n-1)} \leqq (1 - \sum_{r=1}^{n-1} a_{rj})b_{jk}^{(n-1)} \qquad (j \neq k), \tag{2.2}$$

the inequality $\leqq$ in (1.7) becomes strictly $<$. This proves property (I).

However, when $d_n > 0$, it is not necessary that all s_k be less than 1. We wish to analyze the inequality (1.4) further without assuming $s_k < 1$ for all k. Properties (1.2) and (1.3) are assumed in the balance of this section.

(II) *If the* n-1 *column-sums* $s_j = 1$ *for all* $j \neq i$, *then* $b_{ij} = b_{ii}$.

For our proof, we let $i = n$. Replace all the inequalities in (1.6) and (1.7) by equal signs.

(III) *If* $b_{ij} = b_{ii} \neq 0$, *then* $s_j = 1$ *for all* $j \neq i$.

In the proof, we again let $i = n$. Then in (1.7), we have only equal signs. Multiplying both sides by $\delta_{kh} - a_{kh}$ and summing for $k = 1, 2, \ldots, n-1$, we have $a_{nh}d_{n-1} = d_{n-1}(1 - \sum_{k=1}^{n-1} a_{kh})$. Since $d_{n-1} = b_{nn} \neq 0$, we have (III).

(IV) *If* $b_{ii} = 0$ *for some* i, *then* $b_{ij} = 0$ *for all* j, *and* $d_n = 0$.

The proof follows from (1.4) and (1.5)

The contrapositive statement of (III) gives the following fact: If $b_{ii} \neq 0$, and if $s_j < 1$ for some $j \neq i$, then in the ith row of the adjoint, *some* non-diagonal element (we do not know which one) is less than the diagonal element. However, the following statement is more precise:

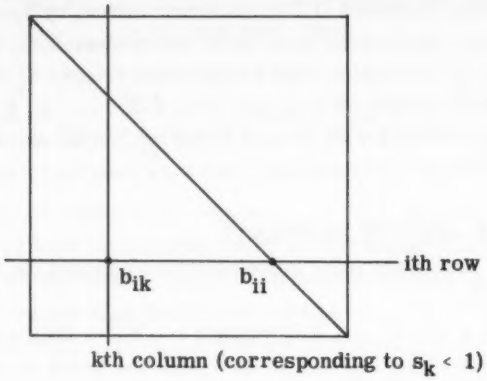
(V) *If* $b_{ii} \neq 0$, *and if there exists* $k \neq i$ *for which* $s_k < 1$, *then* $b_{ik} < b_{ii}$.

The proof consists in showing that the sign $\leqq$ in (1.7) becomes strictly $<$. Note that

$$b_{kk}^{(n-1)} \geqq b_{nn}^{(n)} > 0$$

for $k = 1, 2, \ldots, n-1$. From $s_k < 1$ and $s_j \leqq 1$, we have (2.1) and (2.2). This proves (V).

The following diagram shows property (V) of the adjoint:



(VI) If there exists an integer j for which $s_j < 1$ and $a_{jj} > 0$, then for $i \neq j$,

$$b_{ij} < s_j b_{ii}$$

$$b_{ij} < \frac{s_j - a_{jj}}{1 - a_{jj}} b_{ii} .$$

An elementary proof is given in the author's paper, "An Inequality for Minkowski Matrices," Proceedings of the American Mathematical Society, Vol. 4, February 1953, pp. 137-139 and 141.

Final Remark. If we do not assume property (1.3), then property (1.4) does not hold in general. As a counter-example, let

$$A = \begin{pmatrix} .5 & .2 \\ 1 & .4 \end{pmatrix}.$$

Then I - A has the property that all its principal minors are positive and $d_2 < d_1$. In this example, the input for sector 1 is greater than its output. However, the solution for (1.1) is positive for non-negative h_i (not all zeros). A mathematical treatment for nonnegative matrices with non-dominant diagonal can be given (cf. the author's forthcoming paper on this topic in the Proceedings of the American Mathematical Society).

* * *

DECISION MAKING IN THE FACE OF UNCERTAINTY - II

Richard Bellman
The Rand Corporation

This paper considers multi-stage processes involving both zero-sum and non-zero sum games. Using the concept of "games of survival" approximate solutions are derived for both classes of multi-stage games under various realistic assumptions.

1. INTRODUCTION

In the first paper of this series, [1], we considered a multi-stage decision process of simple and yet general type and showed that an approximate solution of plausible and intuitive sort could be obtained under certain reasonable assumptions.

In this paper, we shall consider a more difficult class of problems, involving conflict between two groups. In many situations of importance, one group may be considered to be the inanimate forces of nature.

When the two groups are in direct conflict we have the theory of zero-sum games to assist us. When the two groups are partly opposed and partly not, we encounter the problems of non-zero sum games, where there is essentially no theory to guide us. Nevertheless, we shall show that under certain conditions we can once again obtain approximate modes of procedure which seem eminently reasonable.

2. DESCRIPTION OF THE MULTI-STAGE PROCESS

We shall consider a multi-stage process of the following type, which we shall call a "game." There are two protagonists, whom we shall call "players," named rather prosaically P and Q. The first of these players, P, has a choice of M different plays, which we shall designate by the numbers 1, 2, . . . , M, and the second player, Q, has a choice of N different plays, denoted by 1, 2, . . . , N. If P chooses the i^{th} of his choices and Q the j^{th} of his choices, then P receives a quantity a_{ij} and Q a quantity b_{ij}. These quantities may be negative, in which case a loss is sustained.

We can now consider this situation to repeat itself for a finite or unbounded number of stages and pose the problem of determining the optimal mode of play for each player under the assumption that each wishes to maximize his over-all return.

3. ZERO-SUM GAMES — SINGLE STAGE

The simplest case, and unfortunately the only case in which a satisfactory theory exists, is that where $b_{ij} = -a_{ij}$, which is to say the players are in direct opposition since one's gain is the other's loss.

We shall begin with the discussion of the single-stage process. It is clear that the determination of optimal play is trivial if either of the players is required to move before the other. The only interesting case is that where both players are required to move simultaneously.

In these circumstances they protect themselves by mixing their choices. Let us assume then that P chooses the i^{th} play with probability x_i and Q his j^{th} choice with probability y_j. The vector $x = (x_1, x_2, \ldots, x_M)$ specifies P's probability distribution, and $y = (y_1, y_2, \ldots, y_N)$ specifies Q's probability distribution.

The expected return for P will be

$$E_P(x,y) = \sum_{i,j=1}^{M,N} a_{ij} x_i y_j , \tag{3.1}$$

and the expected return for Q, $E_Q(x,y)$ will be the negative of this,

$$E_Q(x,y) = -E_p(x,y) . \tag{3.2}$$

The first player, P, will choose his probability distribution x so as to maximize E_P and the second player Q will choose y so as to minimize E_P.

We can then define two values

$$V_P = \operatorname*{Min}_y \operatorname*{Max}_x E_P(x,y) \tag{3.3}$$

and

$$V_Q = \operatorname*{Max}_x \operatorname*{Min}_y E_P(x,y) . \tag{3.4}$$

In each case the variation is over the regions defined by

$$\text{(a)}\ x_i \geq 0, \quad \sum_{i=1}^{M} x_i = 1 ,$$

$$\text{(b)}\ y_j \geq 0, \quad \sum_{j=1}^{N} y_j = 1 . \tag{3.5}$$

The first value, V_P, is the expected return to P if Q is required to choose y before P chooses x, and the second is the expected return to P if P must choose x before Q chooses y.

It is a remarkable fact, the celebrated min-max theorem of Von Neumann, [4], the basic theorem of the theory of games, that

$$V_P = V_Q . \tag{3.6}$$

The interpretation of this result is that P can announce x in advance, and Q likewise, without either gaining from this advance knowledge.

4. ZERO-SUM GAMES — FINITE RESOURCES

In many situations, involving multi-stage play, the above model is not satisfactory. This is particularly true in multi-stage processes where both sides have finite resources. Here the game automatically terminates when either player has no resources.

Let us assume then that each side now plays to ruin the other, with the game continuing until one or the other player is bankrupt. Let p represent the initial amount possessed by P and q the initial amount possessed by Q. We define

(4.1) $$f(p,q) = \text{the probability that P survives Q when P starts with p, Q with q and both sides use optimal play.}$$

Games of this variety are aptly called "games of survival," cf [2], [3].

5. GAMES OF SURVIVAL — MATHEMATICAL FORMULATION

Since P wins what Q loses, and vice versa, the total quantity of resources in the game remains constant, and equal to $p_0 + q_0 = k$, the initial total, to specify the state of the game it is sufficient then to state the amount of resources possessed by P.

We replace f(p,q) by the function of one variable f(p). Let us now derive a functional equation for f(p). Enumerating the possible outcomes of one stage of play, we see that

(5.1) $$f(p) = \sum_{i,j=1}^{M,N} x_i y_j f(p+a_{ij}),$$

where $x = (x_1, x_2, \ldots, x_M)$ and $y = (y_1, y_2, \ldots, y_N)$ are the probability distributions of P and Q determining the initial play.

Since P plays to maximize f(p) and Q to minimize f(p), we obtain the equation

(5.2) $$f(p) = \operatorname*{Max}_x \operatorname*{Min}_y \sum_{i,j=1}^{M,N} x_i y_j f(p+a_{ij}) = \operatorname*{Min}_y \operatorname*{Max}_x \sum_{i,j=1}^{M,N} x_i y_j f(p+a_{ij}).$$

This equation is valid for $0<p<k$. We also have

(5.3) $$f(p) = 1, \quad p \geq k$$
$$f(p) = 0, \quad p \leq 0.$$

6. APPROXIMATE SOLUTION

A solution of (5.2) yields f(p) and thus the vector x(p), y(p), the probability distributions determining the initial play. We wish to determine approximate values for x(p) and y(p) under certain assumptions concerning the sequence of pay-offs, (a_{ij}).

We assume that no single play creates any appreciable change in the state of the game, which is to say that a_{ij} is small compared to p.

We then write

$$f(p+a_{ij}) = f(p) + a_{ij}f'(p). \tag{6.1}$$

Then (5.2) takes the form

$$\begin{aligned} f(p) &\cong \operatorname*{Max}_x \operatorname*{Min}_y \Big[\sum_{i,j=1}^{M,N} x_i y_j \left(f(p) + a_{ij}f'(p)\right) \Big] \\ &\cong \operatorname*{Min}_y \operatorname*{Max}_x \Big[\sum_{i,j=1}^{M,N} x_i y_j \left(f(p) + a_{ij}f'(p)\right) \Big], \end{aligned} \tag{6.2}$$

or

$$\begin{aligned} 0 &\cong \operatorname*{Max}_x \operatorname*{Min}_y \Big[f'(p) \sum_{i,j=1}^{M,N} a_{ij}x_i y_j \Big] \\ &\cong \operatorname*{Min}_y \operatorname*{Max}_x \Big[f'(p) \sum_{i,j=1}^{M,N} a_{ij}x_i y_j \Big]. \end{aligned} \tag{6.3}$$

Let us now assume further that, as is true in all realistic situations, it definitely pays P to have a larger initial quantity of resources, that is to say $f'(p) > 0$ for all p.

Then (6.3) is replaced by

$$\begin{aligned} 0 &\cong \operatorname*{Max}_x \operatorname*{Min}_y \Big[\sum_{i,j=1}^{M,N} a_{ij}x_i y_j \Big] \\ &\cong \operatorname*{Min}_y \operatorname*{Max}_x \Big[\sum_{i,j=1}^{M,N} a_{ij}x_i y_j \Big]. \end{aligned} \tag{6.4}$$

The meaning of this equation is that for large p, with a large number of plays remaining until the end of the game, the play is approximately the same as that employed in the single-stage process where both players wish merely to maximize the expected return from one play.

This approximate solution has precisely the same structure as that given for the one-person process in [1].

The important feature of (6.4) is that we obtain the same approximate equation regardless of the significance of f(p). Consequently, even in situations where f(p) is not completely determined, as frequently occurs in realistic situations, we know that we possess a good approximation to optimal play.

7. NON-ZERO SUM GAMES — GAMES OF SURVIVAL

Let us now turn to a discussion of the more general situation where $b_{ij} \neq -a_{ij}$. Here there is no theory for the determination of optimal play in a single-stage process. Consequently, we shall turn immediately to the discussion of multi-stage processes. We assume once more that both players strive to ruin the other and continue the game until this occurs.

Let p be the initial amount possessed by P and q the amount possessed by Q, and introduce the function defined by (4.1).

This function satisfies the functional equation

$$(a)\quad f(p,q) = \operatorname*{Max}_{x}\operatorname*{Min}_{y} \sum_{i,j=1}^{M,N} x_i y_j f(p + a_{ij}, q + b_{ij})$$

$$= \operatorname*{Min}_{y}\operatorname*{Max}_{x} \sum_{i,j=1}^{M,N} x_i y_j f(p + a_{ij}, q + b_{ij}), \quad p, q > 0, \tag{7.1}$$

$$(b)\quad f(p,q) = 1, \quad p > 0,\ q \leq 0$$

$$= 0, \quad p \leq 0,\ q > 0$$

$$= 1/2, \quad p = q = 0\ .$$

8. APPROXIMATE SOLUTION

Let us assume that a_{ij} and b_{ij} are both negative, so that we are dealing with an attrition process, and that a_{ij} and b_{ij} are small compared to p and q.

We write

$$f(p+a_{ij}, q+b_{ij}) = f(p,q) + a_{ij}f_p + b_{ij}f_q\ . \tag{8.1}$$

Then (7.1a) yields

$$0 \cong \operatorname*{Max}_{x}\operatorname*{Min}_{y} \left[f_p \sum_{i,j=1}^{M,N} a_{ij}x_iy_j + f_q \sum_{i,j=1}^{M,N} b_{ij}x_iy_j \right]$$

$$\cong \operatorname*{Min}_{y}\operatorname*{Max}_{x} \left[f_p \sum_{i,j=1}^{M,N} a_{ij}x_iy_j + f_q \sum_{i,j=1}^{M,N} b_{ij}x_iy_j \right]\ . \tag{8.2}$$

As before we assume that $f_p > 0$, $f_q < 0$. Since $\Sigma\ a_{ij}x_iy_j$, $\Sigma\ b_{ij}x_iy_j$ are both negative, (8.2) yields

$$\frac{f_p}{f_q} \equiv \underset{x}{\text{Max}}\ \underset{y}{\text{Min}} \left[\sum_{i,j=1}^{M,N} b_{ij}x_iy_j \Big/ \sum_{i,j=1}^{M,N} a_{ij}x_iy_j \right]$$

(8.3)

$$\equiv \underset{y}{\text{Min}}\ \underset{x}{\text{Max}} \left[\sum_{i,j=1}^{M,N} b_{ij}x_iy_j \Big/ \sum_{i,j=1}^{M,N} a_{ij}x_iy_j \right].$$

The interpretation of this equation is that both sides play approximately so as to maximize or minimize respectively the ratio

(8.4) $$R(x,y) = \sum_{i,j=1}^{M,N} b_{ij}x_iy_j \Big/ \sum_{i,j=1}^{M,N} a_{ij}x_iy_j.$$

That, under the assumptions on a_{ij} and b_{ij},

(8.5) $$\underset{x}{\text{Max}}\ \underset{y}{\text{Min}}\ R(x,y) = \underset{y}{\text{Min}}\ \underset{x}{\text{Max}}\ R(x,y)$$

is a theorem also due to Von Neumann, and recently established in a different manner by Shapley, [5].

9. A RATIONALE FOR NON-ZERO SUM GAMES

The importance of the above result resides in the fact that it furnishes us a motive for using R (x,y) as a universal pay-off function for non-zero sum games. Whether or not it is to be accepted in any particular situation will depend on other properties of the game.

REFERENCES

[1] Bellman, R., "Decision-Making in the Face of Uncertainty – I," Naval Research Logistics Quarterly, Vol. 1 (1954), pp. 230-232.

[2] Bellman, R., "An Introduction to the Theory of Dynamic Programming," RAND Report No. R-245, 1953.

[3] Bellman, R. and LaSalle, J., "Non-Zero Sum Games and Stochastic Processes" RAND Memorandum 212, 1949.

[4] Von Neumann, J. and Morgenstern, O., The Theory of Games and Economic Behavior, Princeton University Press, 1948.

[5] Shapley, L., "Stochastic Games," Proc. Nat. Acad. Sci., Vol. 39 (1953), pp. 1095-1100.

* * *

NEWS AND MEMORANDA

Readers are invited to submit to the Managing Editor items of general interest in the field of logistics.

RADM. R. J. Arnold, SC, USN, Deputy and Assistant Chief of the Bureau of Supplies and Accounts, became Chief of the Bureau of Supplies and Accounts and the Paymaster General of the Navy on October 4, 1954, relieving RADM. M. L. Royar, SC, USN.

On October 4, 1954, VADM. M. L. Royar, SC, USN, became Chief of Naval Material relieving VADM. John E. Gingrich, USN, who retired.

The Office of Naval Research Logistics Project at Princeton University will hold an informal conference on recent developments in the Theory of Games on January 31 and February 1, 1955 in Fine Hall, Princeton University. The conference will be concerned mainly with games with infinitely many moves and n-person games. Additional information can be obtained from Dr. Philip Wolfe, Fine Hall, Box 708, Princeton University, Princeton, N. J.

The newly formed Institute of Management Sciences has published the first issue of its official journal, Management Science. The journal is to be published quarterly with the first issue dated October 1954. The articles in the journal are on the development or application of scientific knowledge relevant to the field of management.

The George Washington University Logistics Research Project was founded in November 1949 and is the largest contractor for logistics research with the Office of Naval Research. The Principal Investigator of the Project is Dr. Max A. Woodbury, and the staff currently consists of ten Research Associates and supporting personnel. Physically, the Project occupies Staughton Hall on The George Washington University campus, where it has the Logistics Computer, the ONR Relay Computer, and a small punched card machine installation.

Under its original Principal Investigator, Dr. C. B. Tompkins, the Project set up five main objectives. These objectives, which also describe current Project efforts, are:

1. The numerical simulation of any military action in terms of logistic support.
2. The improvement of inventory control.
3. The application of automatic calculating equipment to logistics problems, and the design of new equipment or modification of technical procedures to permit logistic calculations with available equipment.
4. The development of a quantitative expression of the degree to which the elements of logistic support contribute to the success of a military action.

5. The acquisition, reproduction, and circulation of information pertaining to logistics.

In connection with its first objective, the Project has completed three sets of broad commodity aggregate planning factors for use in determining overseas shipping requirements for naval units. These factors have been supplied along with instructions for their application relating them to quantitative statements of "standard operating conditions" which the numbers (usages) simulate. This "Three Factor Method of Logistics Planning" has been used by the Naval War College and by the Atlantic Fleet and is currently being adapted for operational use. A second completed task under the first objective involved the reduction of data concerning the shipyard industry for integration into a quantitative description of the national economy which was prepared under the overall supervision of the Bureau of the Budget. Current Project efforts include a study of "shipyard capacity", determination of implications of construction programs for critical items, and, in cooperation with the University Geography Department, the role of environmental factors in resupply requirements. At quite a different level of abstraction, the Project has made contributions to the study of transportation scheduling problems which have led to current work on the "traveling salesman" problem.

On inventory control, descriptions have been made of current procedures and problems experienced at particular naval activities. Attention has also been given to theoretical models. The current emphasis of the Project is toward applications of usage data to the simulation of inventory control systems with the Logistics Computer, in a manner specified in the third objective.

Under its third objective, the Project initiated a request that the Office of Naval Research procure an electronic computer, designed to carry out conveniently and expeditiously, at least on a research basis, many of the routine short calculations with high input volume which occur in logistics calculations. After consultation with various naval bureaus, the Project stated the basic specifications for the machine, with consideration to several specific logistics calculations. As a result of a contract by the Office of Naval Research with Engineering Research Associates, the Logistics Computer was delivered in February 1953. This machine is a special purpose, magnetic drum memory computer which can conveniently and rapidly maintain records of inventory, consumption, expected inventory, or expected consumption of stocks of about 25,000 items. Larger numbers of items can be handled by making two or more machine runs. In addition to applications to inventory control problems for which the computer was specifically intended, several more involved calculations have been performed in studies of transportation problems, production scheduling, probability, and game theory.

Several studies have been conducted and are in various stages of completion under the fourth objective. In this category are work on aspects of replenishment at sea operations and studies of probabilistic models of warfare. In the latter, fundamental lethal rates are specified and consequent probability expressions are used to provide an attrition structure for the "combat". These theoretical studies of warlike situations concern recognition of probabilistic realities in order to obtain both qualitative and quantitative results including sufficient initial conditions for prescribed expectations.

To meet the fifth objective, the Project reproduces and distributes Logistics Papers, which are collections of working papers prepared under various auspices. Also, annual working conferences in logistics are sponsored jointly with the Office of Naval Research. These conferences have helped stimulate widespread interest in inventory control, organization theory, planning factor methods, transportation problems, etc. The Project also conducts a weekly seminar in Washington.

In addition to the above, a considerable number of the supporting personnel are occupied with the Project's Korean Data Collection Program, initiated in June 1952, and currently the Project's largest endeavor. From the beginning, the Project has been concerned with the application of usage data in order to introduce more measurement into logistics and to provide a foundation for scientific procedures. In this connection, the Project undertook the collection of raw data on material usage by a representative number of ships of different types engaged in the Korean campaign. These data are in the form of the last documents generated in the Navy Supply System by a line item before it passes out of the system. These data cover not only material used by the sample ships themselves but also that used by tenders, shore activities, etc., for the benefit of the sample ships. Such data from 1 July 1950 have been collected and are being reduced into a form for automatic handling. The experience of the Project in collecting and handling these data on an electronic computer is providing the necessary techniques for possible use in a data collecting facility within the navy, as an operating unit. Since the summer of 1954, the data have started to be available in quantities sufficient for analysis by automatic means and several types of analyses are underway.

Throughout the entire existence of the Project, active liaison has been maintained with Supply Demand Control Points, the Fleet, the Naval War College, and other naval activities. Energetic, patient, and conscientious attempts on the part of Naval Officers and operating personnel of the Navy on the one hand, and Project researchers on the other hand, to understand each other's language and problems, have greatly increased the utility of the Project and have made material published under the fifth objective, stated above, intelligible to a wide audience.

The Managing Editor wishes to take this opportunity to acknowledge the generous refereeing of papers for Volume 1 by members of the Editorial Staff and by the following individuals: A. J. Hoffman, W. W. Jacobs, Col. W. P. McBride, J. M. McLynn, Martin Shubik, G. L. Thompson, P. S. Wolfe, and Y. K. Wong. The Managing Editor is also grateful to Selda Futrowsky for her excellent office assistance and to D. E. Darr and Leona Snider of the Naval Research Laboratory for taking care of many details concerning the printing of the issues.

* * *

SHIPS, MACHINERY AND MOSSBACKS. The Autobiography of a Naval Engineer. By Harold G. Bowen, Vice Admiral, USN (Retired), Princeton University Press, 1954, 397 pp.

This book tells the story of a Naval Engineer who was one of that group of engineers, scientists and inventors who create the weapons needed to fight a war. This autobiography covers the naval career of Harold G. Bowen from the time he entered the U. S. Naval Academy in 1901 until he retired from the Navy as a Vice Admiral in 1947, a few days short of 46 years. The book highlights the author's version of the fight on the introduction of "High-Pressure, High-Temperature" steam power plants into Naval Vessels and his experiences in operating struck war plants that were taken over by the government under the war powers of the President during World War II. The book is composed of six chapters as follows: Education, Ashore and Afloat; High-Pressure High-Temperature Steam; Diesel Engines; Naval Research Laboratory; Plant Seizure and Operation; Office of Naval Research.

SCIENTIFIC EXPLANATION. A Study of the Function of Theory, Probability and Law in Science. By Richard B. Braithwaite, Cambridge University Press, 1953, 376 pp.

The author examines the logical features common to all the sciences. Every science proceeds by inventing general hypotheses from which are deduced particular consequences to be tested by observation and experiment. Probability statements, their meaning and the role they play in science, are dealt with; and probability mathematics is developed far enough to elucidate the meaning of the statistical hypotheses used in science. The basic concepts of the theory of games of strategy are presented and their significance is discussed in connection with statistical inference. The nature of scientific "models" is carefully described, both regarding their uses and misuses.

SHIPS, SALVAGE, AND SINEWS OF WAR. By Rear Admirals W. R. Carter and E. E. Duvall, U. S. N. (Retired), with a Foreword by Admiral R. B. Carney, U. S. N., and an Introduction by Admiral H. K. Hewitt, U. S. N. (Retired), U. S. Government Printing Office, 1955, 533 pp., including 112 photographs.

This is the story of fleet logistics afloat in Atlantic and Mediterranean waters during World War II. It starts with a discussion of the Neutrality Patrol and Acquisition of Lend Lease Bases; discusses the organization and capabilities of our Atlantic Bases and of the Service Force; gives a chronological account of the logistic support of the Atlantic Fleet in all their operations; and ends with the Rhine River crossings and the German defeat. The book contains details of ships engaged in various operations, the manner of their commanders, and the organization. It covers the quantitative statistics of logistic support in detail. In the author's preface it states: "Source material for this volume has been taken from official naval records, such as war diaries, ship's logs, operation plans, action reports; and from interviews,

and personal letters." This is the second book by Admiral Carter dealing with Fleet Logistics Afloat. The first, Beans, Bullets and Black Oil covered the Pacific Fleet.

SAMPLE SURVEY METHODS AND THEORY. Vol. I-Methods and Applications, 638 pp. Vol. II-Theory, 332 pp. By Morris H. Hansen, William N. Hurwitz, and William G. Madow, John Wiley & Sons, Inc., 1953.

The two volumes together represent an attempt to present a comprehensive textbook and reference manual on sampling theory and practice for the student and for those in charge of the design of sample surveys. The work is also intended for the user of the results of surveys who wishes to know the circumstances under which he may place confidence in information based on samples. The first volume gives the principles and methods of sampling and their applications. Formulas appropriate to the methods are given without proof. The second volume contains the fundamental theory, together with derivations of the formulas and proofs of statements made in Volume I. The level of difficulty has been kept to the minimum; thus, Volume I can be read without any particular mathematical training, while Volume II requires not more than college algebra and only on some occasions simple calculus. The work is provided with a very comprehensive index to facilitate its use as a reference work.

OPERATIONS RESEARCH FOR MANAGEMENT. Edited by Joseph F. McCloskey and Florence N. Trefethen, The Johns Hopkins Press, 1954, 409 pp.

This volume is not a primer on operations research, but is, mainly, a collection of papers presented at various times in seminars sponsored by the Operations Research Office, The Johns Hopkins University. A comprehensive bibliography by Vera Riley is included. As indicated by the title, the editors claim that this book is addressed primarily to management. Part I contains papers on the history of operations research and on the role of operations research as a science, as a profession, and as a method for management consulting. Part II is on methodology and presents expository papers on various techniques employed in operations research, e.g., queueing theory, information theory, linear programming, statistical theory, and game theory. Part III is on case histories and includes papers on research in supermarkets, the printing industry, agriculture, the effect of promotional effort on sales, and the reliability of airborne radar.

LONG RANGE ECONOMIC PROJECTION. Studies in Income and Wealth, Volume Sixteen. By the Conference on Research in Income and Wealth, National Bureau of Economic Research, Princeton University Press, 1954, 476 pp.

This volume contains the papers delivered at the May 1951 meetings of the Conference on Research in Income and Wealth, together with the comments of participants. The contributors to this volume have concerned themselves mainly with the conceptual problems of long-term projections, and the manner in which economic problems are involved. While not completely neglected, the problem of obtaining specific statistical projections is not foremost.

The four parts of the volume are arranged according to the areas of the economy with which they are concerned. Part I contains papers by Kuznets, Wool, and Kendrick, dealing with the problem of long-term projection as a whole, and the problems of manpower and productivity. In Part II, Cavin, Daly, Barnett, and Boschen are concerned with the problems of projecting the growth of various industrial parts of the economy. Part III has papers by Feller, Smelker, Smithies, and Polak, treating projections in different sectors of the economy. Finally, in Part IV Isard and Freutel discuss the area of regional projections.

READINGS IN GAME THEORY AND POLITICAL BEHAVIOR. By Martin Shubik, Doubleday and Company, Inc., 1954, 74 pp.

This book is a brief collection of an introduction and ten readings designed to present individuals in administration and the social sciences, who do not necessarily have a formal training in mathematics, with the basic ideas of game theory. The introduction and most of the articles are entirely non-mathematical and use no symbols whatsoever. The article by Abraham Wald, which is a review of the book, Theory of Games and Economic Behavior, and the article by Jacob Marschak, which deals with the problem of constructing a preference scale for decision-making, can be followed by the non-mathematician but need careful reading. Another article by John McDonald and John W. Tukey gives an example of a simple strategic situation in which the computations required for the "Colonel Blotto" tactical problem are carried out.

DECISION PROCESSES. Edited by R. M. Thrall, C. H. Coombs, and R. L. Davis, John Wiley & Sons, Inc., 1954, 332 pp.

This volume contains a collection of papers which resulted from the University of Michigan seminar of 1952, held at Santa Monica, California, on The Design of Experiments in Decision Processes. The 19 chapters, written by twenty-four contributors, are directed at the application of mathematics to the behavioral sciences, with particular emphasis on decision processes. After an introductory chapter and one on Mathematical Models and Measurement Theory, the remaining chapters are divided into four groups with the headings: Individual and Social Choice, Learning Theory, Theory and Applications of Utility, and Experimental Studies.

* * *

oU. S. GOVERNMENT PRINTING OFFICE : 1955 O - 346750